Definition

If a person A borrows some money from another person B for a certain period, then after that specified period, the borrower has to return the money borrowed as well as some additional money. This additional money that borrower has to pay is called *interest*. The actually borrowed money by A is called **principal** (SUM). The principal and the interest together is called *amount*. The interest that the borrower has to pay for every 100 rupees borrowed for every year is known as **rate per cent per annum**. It is denoted as R% per annum $= \dfrac{R}{100}$.

The time for which the borrowed money has been used is called the *time*. It is denoted as T years. The interest is directly proportional to the principal, the rate and time for which the borrowed sum is used.

If the interest on a certain sum borrowed for a certain period is reckoned uniformly, then it is called **Simple Interest** and denoted as S.I.
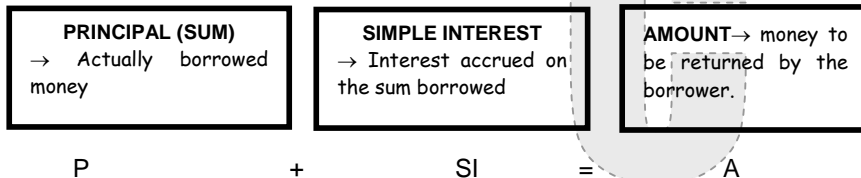
$\therefore$ Simple Interest (S.I.) $= \dfrac{P \times R \times T}{100}$

Where P = Principal or the sum borrowed

R = Rate per cent per annum

T = Number of years for which the borrowed money has been used.

Amount

PRINCIPAL (SUM)	SIMPLE INTEREST	AMOUNT$\rightarrow$ money to
$\rightarrow$ Actually borrowed money	$\rightarrow$ Interest accrued on the sum borrowed	be returned by the borrower.

P + SI = A

$\therefore$ A = P + SI = P + $\dfrac{PRT}{100}$ or

$$A = P\left[1+\dfrac{RT}{100}\right]$$

Ex.1 *A certain sum of money invested at some rate of interest triple it self in 4 years. In how many years the principal will become 9 times of itself at the same rate?*

Sol. When the principal is in simple interest the interest for every year will be same. In 3 years the amount becomes 3 times the principal and we have

A = P + I. or 3P = P + I $\Rightarrow$ I = 2P

i.e. the interest is 2 time the principal in 4 years or equal to principal in 2 years.

The interest will be equal to P in 2 years. So interest will be 8P in 16 years.

Amount after 16 years = P + 8P = 9P.

Hence the required answer will be 16 years

REPAYMENT OF DEBT IN EQUAL INSTALLMENTS:

Ex.2 *Dr. M. I. Rajpoot wants to buy an air-conditioner for his family. He went to a shop and selects an AC whose price is Rs. 12,700. The shopkeeper gave him two offers either you pay full amount of Rs. 12,700 or pay only Rs. 4,000 and installment of Rs. 3000 per month for next 3 months just paying only Rs. 300 as interest. What is the rate of interest the shop keeper charged to Rajpoot?*

Sol. In these types of problems the interest charged is always calculated on the basis of the one month principal not on the amount of the loan taken. Here first of all we have to calculate the one month principal for every installment paid.

Principal for 1st month = loan amount = Rs 8,700

Principal for 2nd month = loan amount – 1st installment = Rs (8,700 – 3,000) = Rs. 5,700

Principal for 3rd month = Rs. (5,700 – 3,000) = Rs. 2,700

Total one month principal = Rs.(8700 + 5700 + 2700) = Rs. 17,100

Time = $\dfrac{1}{12}$ years, Interest = Rs. 300, Rate =?, Principal = Rs. 17,100

Using the formula

$I = \dfrac{PRT}{100} \Rightarrow 300 = \dfrac{17100 \times R \times 1}{100 \times 12} = 21.05\%$

✎ **Toolkit**

> If M amount is taken for n months at a simple interest of r% and installment paid per month is "a". Then we can find the required value with the help of following formula
>
> $$na - M = \left[nM - \frac{a(n-1)n}{2} \right] \frac{r}{100} \times \frac{1}{12}$$

Compound Interest

As discussed in the topic on 'Simple Interest', the principal (P) remains constant throughout the period for which the money (principal) is borrowed. But, in case of compound interest, the total interest received in the present year will be added to the original principal and for the following year the principal will be Amount received (Principal + interest).

(a) $A = P\left[1 + \dfrac{R}{100}\right]^{n}$ (Compounded Annually)

$A = P\left[1 + \dfrac{R}{2 \times 100}\right]^{2n}$ (Compounded Half- yearly)

$A = P\left[1 + \dfrac{R}{4 \times 100}\right]^{4n}$ (Compounded Quarterly)

Where, R = rate per cent year (% p.a.)

n = time in year,

A = Amount

(b) Compound Interest (CI) = A – P

$$C.I = P\left[\left(1+\frac{R}{100}\right)^t - 1\right]$$

Ex.3 *A certain sum of money doubles in 3 years, then in how many years it will become 8 times at compound interest..*

Sol. Think logically that in every 2 years, the principal becomes doubles of itself. So in 4 years it will be 4 times and in next two years it will be double of 4 times that is 8 times of original principal. So the required answer would be 8 years.

Difference between CI & SI

For the first year the simple interest and compound interest, both are same, but with the next following years the C.I will always be more than the S.I. Let P is the principal invested at r% rate per annum at S.I and C.I respectively. What will be the difference between SI and CI for different years?

Years	Simple Interest	Compound Interest	Difference
After 1 year	$\frac{Pr}{100}$	$\frac{Pr}{100}$	0
After 2 year	$\frac{Pr}{100}$	$\frac{Pr}{100} + P\left(\frac{r}{100}\right)^2$	$P\left(\frac{r}{100}\right)^2$
After 3 year	$\frac{Pr}{100}$	$\frac{Pr}{100} + 2P\left(\frac{r}{100}\right)^2 + P\left(\frac{r}{100}\right)^3$	$3P\left(\frac{r}{100}\right)^2 + P\left(\frac{r}{100}\right)^3$

TIP

SI remains constant every year as the principal remains the same every year.

CI increases every year as principal increases every year.

Hence for every changing year, the principal goes on changing and accordingly the amount of interest accrued on varying principal will be different in every year. The money lent under this condition is charged with **Compound Interest**.

While solving the problems on Compound Interest, it is assumed that Interest is compounded yearly, unless otherwise specified.

Equal annual installment

Let the value of each equal annual installment = Rs. A

Rate of interest = R % p.a. at CI

Number of installments per year = n

Number of years = T

∴ Total number of installments = n × T

Borrowed Amount = B

Then,

$$A\left[\frac{100}{100+R} + \left(\frac{100}{100+R}\right)^2 + \dots\dots + \left(\frac{100}{100+R}\right)^{n\times T}\right] = B$$

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Ex.4 **If a principle becomes 3 times in 10 years according to simple interest, what is the rate of interest?**

Sol. Principle becomes 3 times means, the interest is 2P. $\therefore 2P = \dfrac{P \times 10 \times r}{100} \Rightarrow r = 20\%$

Ex.5 **A man took Rs. 5000 at 10% simple interest and gave it to another person at 10% compound interest, which is being compounded annually. After 3 years, how much extra money he will get?**

Sol. He has to pay $5000 + \dfrac{5000 \times 10 \times 3}{100} = 6500$

He will get $5000\left(1 + \dfrac{10}{100}\right)^3 = 6655$

So, the ans is 6655 − 6500 = **Rs 155.**

Ex.6 **Certain money becomes double in 4 years according to simple interest. In how many years, will it becomes 3 times?**

Sol. If P is the principle, it becomes 2P in 4 years, that means the interest earned is 2P − P = P.
If it has to become 3 times means the interest has to become 2P, so, it takes double the time.
i.e. 2 × 4 = **8 years**.

Ex.7 **Certain money doubles itself according to compound interest in 3 years. How much time it takes to become 4 times?**

Sol. Since the money gets doubled in 3 years according to the Compound Interest, it again gets doubled in the next 3 years. So it gets 4 times in 6 years.
$\therefore$ **n = 6 years.**

Ex.8 **Mr. A deposited certain amount in a bank exactly 5 years back @ of 8.8% simple interest. Mr. B deposited the same amount of money exactly 2 years back, which is for compound interest, compounded annually. Now both they got same amount of money at what rate of interest, Mr. B deposited his money.**

Sol. Mr. A will get $P + \dfrac{P \times 5 \times 8.8}{100} = 1.44P$

Mr. B will get $P\left(1 + \dfrac{r}{100}\right)^2$

Since both got the same money.

$P\left(1 + \dfrac{r}{100}\right)^2 = 1.44P$

$\Rightarrow$ **r = 20%**

Ex.9 **On a sum of Rs. 1000, the C.I. for 2 years is twice the S.I. for 2 years when the rate is 11%. Find the rate at which the interest is compounded annually?**

Sol. S.I. at 11% for 2 years on a sum of Rs. 1000 = 220
Since C.I. is twice the S.I.

$$\therefore 440 = 1000\left(1+\frac{R}{100}\right)^2 - 1000$$

$$\Rightarrow \frac{1440}{1000} = \left(1+\frac{R}{100}\right)^2$$

$$\Rightarrow 1.2 = \left(1+\frac{R}{100}\right) \Rightarrow R = 20\%$$

Ex.10 Two equal sums are lent at the same time at 6% and 5% simple interest respectively. The former is received 2 years earlier than the later, and the amount in each case is Rs. 2400. Find the sum?

Sol. Let the sum be Rs. x & the latest period be n years

$$x + \frac{x(6)(n-2)}{100} = 2400 \qquad (I)$$

$$x + \frac{x(5)(n)}{100} = 2400 \qquad (II)$$

Solving these two equations simultaneously,

n = 12, x = 1500   The periods are 10 years & 12 years, Sum is Rs. 1500.

Ex.11 Ram invests a certain amount of money and earns a Compound Interest of Rs. 420 in the second year and a C.I. of Rs. 462 in the third year. Calculate at what rate of interest did Ram invest?

Sol. C.I. on third year – C.I. on second year = 462 – 420 = 42
Thus Rs. 42 is the interest on Rs. 420.
i.e. 10% of 420   Hence Rate = 10%.

Ex.12 Robin lend out Rs. 9 on the condition that the loan is payable in 10 months by 10 equal installments of Re. 1 each. Find the rate of interest per annum.

Sol. Let the rate of interest per month be r
Total amount repaid = Rs.10 interest = Re. 1

$$\frac{r}{100}(9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1) = 1$$

$$r = \frac{100}{45}$$

Rate of interest per annum = (100/45) 12 = 26 2/3%.

Shares

The capital is divided into equal parts called **shares**. Shares may be of any value from Rs. 10 (or less) up to Rs. 100 (or more). Each person who purchases shares is called **share holder** and becomes a part owner of the company. He gets a **share certificate** indicating the number of shares he holds in the company. Sometimes a company issues a certificate called **the stock certificate** instead of share certificate indicating that a person holds a stock of say Rs. 5000 i.e. shares worth Rs. 5000. The value of a share mentioned in the share certificate is called its **nominal value, face value or par value** (nominal value does not indicate its cash value). The nominal value is a fixed quantity, but the cash value may vary from hour to hour. The price of a share at any particular time is called its **market value**. Shares can be **preferred shares** or **ordinary shares**.

> **Do you know?**
>
> **(i)** Stock of a company is the capital of the company that can be divided in shares or no. of shares multiplied by the face value of the share.
>
> **(ii)** Dividend is always on the face value and Income is always on the market value of the share.

Stock

The term stock is also applied to the various amounts of money which have been borrowed by Government or by the corporations of cities. To pay the interest on different sums of money borrowed from time to time, the Government sets aside certain funds.

Kinds of Stock

(1) **Stock at Par:** Stock is said to be **at par** if the market value of the stock is equal to its face value. **(Market Value = Face Value)**

(2) **Stock at a premium (or above par):** Stock is said to be at a **premium** if the market value of the stock is greater than its face value. **(Market value = Face value + Premium)**

(3) **Stock at a discount (or below par):** Stock is said to be at a **discount** if the market value of the stock is less than its face value. **(Market value = Face value – discount)**

For example, If Rs. 100 stock is bought for Rs. 106, the stock is at a **premium** of 6% or 6 above par; but if it is bought for Rs. 96, the stock is at a **discount** of 4% or 4 below par; and if Rs. 100 stock is bought for Rs. 100, it is said to be **at par**.

> **TIP**
> **(1)** If stock is given, convert it into cash.
> **(2)** If cash is given, convert it into stock.

Distinction between stock & shares

The **total investment** of a company is called stock. This stock is divided into a number of shares of equal face value. The total face value of a number of shares is also stock. For example, 100 shares of Rs. 10 each is Rs. 1000 stock and 10 shares of Rs. 100 each is also Rs. 1000 stock. Clearly **a share is also a stock, but a stock may not be a share**, for it may consist of more than one share.

Dividend

When a company makes a profit, part of that profit is divided amongst the shareholders and it is called the **dividend**. Dividend is always calculated on the face value of a share and is generally expressed as percentage.

Brokerage

The stock is generally bought or sold through a broker who charges a small commission called the **brokerage**. A buyer has to pay the market value together with the brokerage and a seller gets market value reduced by the brokerage i.e.

> **TIP**
> **Rule:** Add brokerage while buying and subtract while selling.

Amount paid by the buyer = Market value + Brokerage.
Amount received by the seller = Market value – Brokerage.

Debentures

A debenture is an acknowledgement of a debt of a company. Debentures are equal parts of a loan raised by a company. A debenture holder is a creditor of the company and is entitled to a fixed return every year irrespective of profits. If a company is wound up, the amount due to debenture holders must be paid before anything is paid to the share holders.

Ex.13 *How much stock can be purchased with Rs. 52,625 at 5% above par? (Brokerage ¼ %)*

Sol. To purchase Rs. 100 stock we need Rs. $\left(100 + 5 + \dfrac{1}{4}\right)$ = Rs. $\dfrac{421}{4}$.

If investment is Rs. $\dfrac{421}{4}$; stock purchased = Rs. 100.

If investment is Rs. 52,625; stock purchased = $100 \times \dfrac{4}{421} \times 52,625$ = Rs. 50,000.

Ex.14 *A man buys Rs. 6000 stock at 5% discount and sells at 2% above par. Find his gain or loss.*
Brokerage @ $\dfrac{1}{2}$ %.

Sol.
Purchase Price of Rs. 100 stock	= 100 – 5 = Rs. 95
Net purchase price of Rs. 100 stock	= 95 + $\dfrac{1}{2}$ = Rs. $\dfrac{191}{2}$
Net purchase price of Rs. 6000 stock	= $\dfrac{191}{2} \times \dfrac{6000}{100}$ = Rs. 5730
Sale price of Rs. 100 stock	= 100 + 2 = Rs. 102
Net sale price of Rs. 100 stock	= 102 – $\dfrac{1}{2}$ = Rs. $\dfrac{203}{2}$
Net sale price of Rs. 6000 stock	= $\dfrac{203}{2} \times \dfrac{6000}{100}$ = Rs. 6090
Net gain	= 6090 – 5730 = Rs. 360.

Alternate Method: From above –

Gain on Rs. 100 stock   = $\dfrac{203}{2} - \dfrac{191}{2}$ = Rs. 6

Gain on Rs. 6000 stock   = $\dfrac{6}{100} \times 6000$ = Rs. 360.

Income Problem

A statement such as '5% stock @ 95' means.
1.   Face value of stock is Rs. 100.
2.   Market value of Rs. 100 stock is Rs. 95.
3.   Income (Annual) from this stock is Rs. 5.
Hence we get Rs. 5 as income or dividend (Annual) by investing Rs. 95 and obtain a stock worth Rs. 100.

NOTE: Percentage Return on the shares purchased less than the market value will always be more than the actual return.

Ex.15 *X invested an amount of Rs. 23,920 in 8% stock at 92. Find his net income if he pays 4% income tax.*

Sol. Cost of Rs. 100 stock = Rs. 92

Income on Rs. 92 = Rs. 8

Income tax $= \dfrac{8 \times 4}{100} =$ Rs. $\dfrac{8}{25}$

Net income $= 8 - \dfrac{8}{25} =$ Rs. $\dfrac{192}{25}$

Now if amount invested Rs. 92, net income = Rs. $\dfrac{192}{25}$

If amount invested Rs. 23,920, net income $= \dfrac{192 \times 23920}{25 \times 92} =$ Rs. 1996.80.