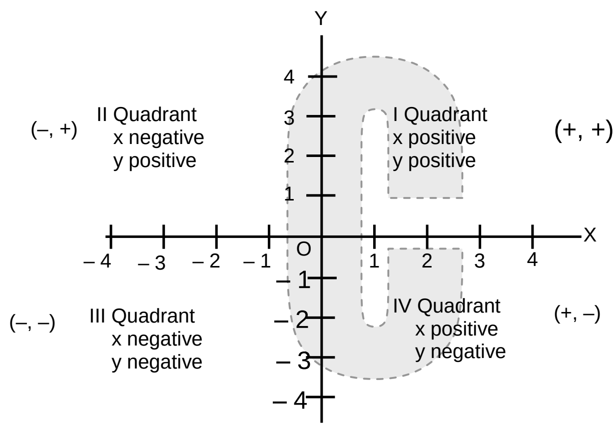
The X – Y Plane

The number lines, when drawn as shown in X – Y plane below, are called "**axes**". The horizontal number line is called the "***x*-axis**", the vertical one is the ***y*-axis**.

Abscissa is the x–coordinate of a point can be defined either as its distance along the x–axis, or as its perpendicular distance from the y–axis.

Ordinate is the y-coordinate of a point can be defined either as its distance along the y-axis, or as its perpendicular distance from the x-axis.

In the figure given below, OX and OY are two straight lines which are perpendicular to each other and which intersect at the point O. OX is known as the x–axis, and OY is known as the y–axis. You can see that the two axes divide the plane into four regions as above. The four regions are known as **Quadrants** and are named I Quadrant, II Quadrant, III Quadrant and IV Quadrant as shown.



(A) Co-ordinate of the origin is (0, 0).

(B) Any point on the x axis can be taken as (a, 0)

(C) Any point on the y axis can be taken as (0, b)

Ex.1 In which quadrant is (*x*, *y*), such that *x y* < 0?

Sol. The points (x, y), with xy < 0 means the product of abscissa and ordinate should be negative which can be possible only when one is positive and other is negative, such as (– 2, 5) or (4, – 6) and this will lie in the Quadrants II and IV.

(i) **Distance formula:**

If A (x_1, y_1) and B (x_2, y_2) be two points, then

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

In particular, of a point P (x, y) from O (0, 0) is $|OP| = \sqrt{x^2 + y^2}$

Ex.2 A (a, 0) and B (3a, 0) are the vertices of an equilateral triangle ABC. What are the coordinates of C?

(1) (a, a√3) (2) (a√3, 2a) (3) (a√3, 0) (4) (2a, + a√3) (5) None of these

Sol. Length AB = $\sqrt{(a-3a)^2}$ = 2a

Now, the vertex C will be such that AC = BC = 2a

$\therefore$ if (x, y) are the co-ordinates of C

$\sqrt{(a-x)^2+(0-y)^2} = \sqrt{(3a-x)^2+(0-y)^2} = 2a$

or $a^2 + x^2 - 2ax + y^2 = 9a^2 + x^2 - 6ax + y^2 \Rightarrow 4ax = 8a^2$

or x = 2a.

$\therefore$ length AC = $\sqrt{(a-x)^2+y^2}$ = 2a

$\Rightarrow \sqrt{a^2+y^2} = 2a$ or $a^2 + y^2 = 4a^2$

$\Rightarrow y^2 = 3a^2$

$\therefore y = \pm\sqrt{3}$ a.

Thus, C is $\left(2a, \pm\sqrt{3a}\right)$.

(ii) **Section formula:**

The point which divides the join of two distinct points A (x_1, y_1) and B (x_2, y_2) in the ratio $m_1 : m_2$ Internally, has the co-ordinates

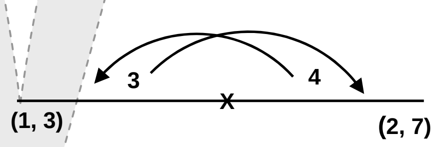
$\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$, $m_1 \neq 0$, $m_2 \neq 0$, $m_1 + m_2 \neq 0$ and externally, is

$\left(\frac{m_1x_2-m_2x_1}{m_1-m_2}, \frac{m_1y_2-m_2y_1}{m_1-m_2}\right)$ $m_1 \neq 0$, $m_2 \neq 0$, $m_1 - m_2 \neq 0$

In particular, the mid-point of the segment joining A $(x_1\ y_1)$ and B (x_2, y_2) has the co-ordinates $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Ex.3 Find the points A and B which divide the join of points (1, 3) and (2, 7) in ratio 3 : 4 both internally & externally respectively .

Sol. Internally: $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$



i.e. $\left(\frac{3\times2+4\times1}{3+4}, \frac{3\times7+4\times3}{3+4}\right) \Rightarrow \left(\frac{10}{7}, \frac{33}{7}\right)$

Externally: $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$

i.e. $\left(\frac{3\times2-4\times1}{3-4}, \frac{3\times7-4\times3}{3-4}\right) \Rightarrow (-2, -9)$.

(iii) **Centroid and Incentre formulae:**

Centroid: It is the point of intersection of the medians of a triangle.

Incentre: It is the point of intersection of the internal angle bisectors of the angles of a triangle.

If A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) be the vertices of a triangle, then its **centroid** is given by $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$, and the incentre by, $\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$.

Where a = | BC |, b = | CA | and c = | AB |.

Ex.4 If (2, 3), (3, a), (b, – 2) are the vertices of the triangle whose centroid is (0, 0), then find the value of a and b respectively.

(1) – 1, – 4 (2) – 2, – 5 (3) – 1, – 6 (4) – 1, – 5 (5) None of these

Sol. x co-ordinate of the centroid = $\frac{2+3+b}{3}$

$$0 = \frac{5+b}{3} \Rightarrow b = -5$$

y co-ordinate of the centroid = $\frac{3+a+(-2)}{3}$

$$0 = \frac{3+a-2}{3} \Rightarrow 0 = a + 1$$

a = – 1 **Answer: (4)**

(iv) **Area of triangle:**

If A (x_1, y_1), B (x_2, y_2) and C(x_3, y_3) are the vertices of a triangle then its area is equal to $= \frac{1}{2}$ mod of $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$= \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

$$\frac{1}{2}[y_1(x_2-x_3)+y_2(x_3-x_1)+y_3(x_1-x_2)]$$

FACT

Three points are collinear if ; the area of the triangle formed by them is zero.

Ex.5 ***A triangle has vertices A (2, 2), B (5, 2) and C (5, 6). What type of triangle it is ?***

Sol. By the distance formula

d (AC) = $\sqrt{(5-2)^2+(6-2)^2}$

$= \sqrt{(3)^2+(4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

d (AB) = $\sqrt{(5-2)^2+(2-2)^2}$

$= \sqrt{(3)^2} = \sqrt{9} = 3$

and d (BC) = $\sqrt{(5-5)^2+(2-6)^3}$

$= \sqrt{(-4)^2} = \sqrt{16} = 4$

$\therefore$ d(AC) = 5, d (AB) = 3, and d (BC) = 4

According to Pythagorean if the sum of the square of two sides are equal to the square of the third side then the triangle is a right-angled triangle.

$d(AC)^2 = d(AB)^2 + d(BC)^2$

$(5)^2 = (3)^2 + (4)^2$

Therefore, the triangle is right-angled.

Important

In question relating to geometrical figures, take the given order and proceed as indicated.	
For an isosceles triangle	Prove that at least two sides are equal.
For an equilateral triangle	Prove that three sides are equal.
For a right-angled triangle	Prove that the sum of squares of two sides is equal to the square of the third side.
For a square	Prove that the four sides are equal, two diagonals are equal.
For rhombus	Prove that four sides are equal (and there is no need to establish that two diagonals are unequal as the square is also a rhombus.)
For a rectangle	Prove that the opposite sides are equal and two diagonals are equal.
For a parallelogram	Prove that the opposite sides are equal (and there is no need to establish that two diagonals are unequal as a rectangle is also a parallelogram.)
For three points to be collinear	Prove that the area of the triangle formed by them is zero.

Slope of a line

First, talk in intuitive terms about what is meant by slope. Give real-life examples of slope such as the slope of the roof of a house, a road going up a hill, or a ladder leaning against a building. Explain that we can assign a number that allows us to measure the *steepness* of a straight line. Also, say that the greater the absolute value of this number, the steeper the line will be.

Slope of a non-vertical line L is the tangent of the angle θ, which the line L makes with the positive direction of x-axis. In particular,

(a) Slope of a line parallel of x-axis is zero.

(b) Slope of a line parallel to y-axis is not defined.

(c) Slope of a line equally inclined to the axis is –1 or 1.

(d) Slope of a line making equal intercepts on the axis is –1.

(e) Slope of the line through the points A (x_1, y_1) and B (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$

(f) Slope of the line $ax + by + c = 0$, $b \neq 0$, is $-\frac{a}{b}$.

(g) Slopes of two parallel (non-vertical) lines are equal. If m_1, m_2 are the slopes, then $m_1 = m_2$.

(h) If m_1 and m_2 be the slopes of two perpendicular lines (which are oblique), then $m_1 m_2 = -1$.

Straight line

Straight-line equations, or "linear" equations, graph as straight lines, and have simple variables with no exponents on them. If you see an equation with *x* and *y*, then you're dealing with a straight-line equation.

An equation of the form ax + by + c = 0 is called the general equation of a straight line, where x and y are variable and a, b, c are constants.

Equation of a line parallel to X axis or Y - axis

(i) Equation of any line parallel to x-axis is y = b, b being the directed distance of the line from the x-axis. *In particular equation of x-axis is y = 0*

(ii) Equation of any line parallel to y-axis is x = a, a being the directed distance of the line from the y-axis. *In particular equation of y-axis is x = 0.*

(a) One point form

Equation of a line (non-vertical) through the point (x_1, y_1) and having slope m is

$\mathbf{y - y_1 = m (x - x_1)}$.

(b) Two-point form

Equation of a line (non-vertical) through the points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1), x_1 \neq x_2$$

(c) Slope-intercept form

Equation of a line (non-vertical) with slope m and cutting off an intercept c from the y-axis is

$$\mathbf{y = m x + c}.$$

(d) Intercept form

Equation of a line (non-vertical) with slope m and cutting off intercepts a and b from the x-axis and y-axis respectively is $\frac{x}{a} + \frac{y}{b} = 1$.

TIP

☞The equation of any line parallel to ax + by + c = 0 is of the type ax + by + k = 0, i.e., there is change only in constant term.

☞The equation of any line perpendicular to ax + by + c = 0 is of the type bx – ay + k = 0 i.e., interchange the coefficient of x and y and change the constant term.

Ex.6 Line intersects x axis at A (10, 0) and y-axis at B (0, 10). Find the equation of the line.

(1) x + y = 10 **(2) x + y = 20** **(3) x = – y** **(4) None of these**

Sol. As line intersects x-axis at A (10, 0)

$\Rightarrow$ length of intercept on x-axis, a = 10

Similarly length of intercept on y-axis, b = 10

$\therefore$ Using intercept form, equation of line is $\frac{x}{10} + \frac{y}{10} = 1$

or x + y = 10. **Answer: (1)**

Ex.7 Find the equation of the straight line passing through the point (– 2, – 3) and perpendicular to the line through (– 2, 3) and (– 5, – 6).

(1) X + 2 Y + 8 = 0 (2) X + 3Y + 11 = 0 (3) X – 3Y = 7 (4) X + 3Y = 11

Sol. The slope of the line through (– 2, 3) and (– 5, – 6) is m = $\frac{-6-3}{-5+2}$ = 3

$\Rightarrow$ The slope m_1 of the required line = – $\frac{1}{3}$.

By point – slope form, Y + 3 = – $\frac{1}{3}$ (X + 2) $(\because m_1 m_2 = -1)$

$\Rightarrow$ X + 3Y + 11 = 0. **Answer: (2)**

Ex.8 Find the slope of the line passing through (– 3, 7) having Y-intercept – 2.

(1) – 5 (2) 2 (3) – 3 (4) $\frac{3}{2}$

Sol. The line passes through the points (– 3, 7) and (0, – 2).

$\therefore$ Slope of the line = $\frac{-2-7}{0+3}$ = – 3. **Answer: (3)**

Some Important Results

- Length of perpendicular from the point (x_1, y_1) to the line ax + by + c = 0 is

$$L = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

- Distance between parallel lines ax + by + c = 0 and ax + by + d = 0

$$\frac{|c - d|}{\sqrt{a^2 + b^2}}$$

- The angle between two lines y = m_1x + b_1 and y = m_2x + b_2 is given by

$$\tan\theta = \pm\left(\frac{m_2 - m_1}{1 + m_1 m_2}\right),\ m_1 m_2 \neq -1$$

- The equation a_1x + b_1y + c_1 = 0 and a_2x + b_2y + c_2 = 0 represent the same line if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Concurrent Lines:

Three or more lines are said to be concurrent lines when all of them pass through a common point.

Standard form of Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

Where centre is (h, k) and radius is ‘r’

If centre of the circle is at the origin and radius is 'r', then equation of the circle is

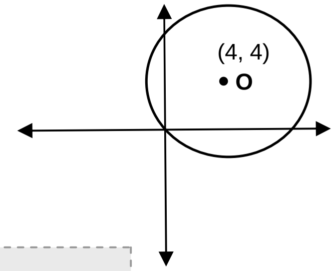
$$x^2 + y^2 = r^2$$

This is also known as the **simplest form.**

Ex.9 If the area of the circle shown below is $k\pi$, what is the value of k?

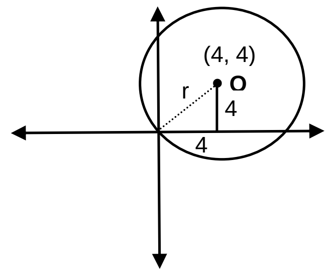
(1) 4 **(2) 16**

(3) 32 **(4) 20**



Sol. Since the line segment joining (4, 4) and (0, 0) is a radius of the circle, $r^2 = 4^2 + 4^2 = 32$.

Therefore, area = $\pi r^2 = 32\pi \Rightarrow k = 32$. **Answer: (3)**



The Ellipse

Though not so simple as the circle, the ellipse is nevertheless the curve most often "seen" in everyday life. The reason is that every circle, viewed obliquely, appears elliptical.

The equation of an ellipse centered at the origin and with axial intersection at ($\pm$, a, 0) and (0, $\pm$ b) is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The Parabola

When a baseball is hit into the air, it follows a parabolic path.

There are all kinds of parabolas, and there's no simple, general parabola formula for you to memorize. You should know, however, that the graph of the general quadratic equation $y = ax^2 + bx + c$ is a parabola. It's one that opens up either on top or on bottom, with an axis of symmetry parallel to the y-axis. The graph of the general quadratic equation $\mathbf{y = ax^2 + bx + c}$ is a parabola. Here, for example are parabolas representing the equations; $\mathbf{y = x^2 - 2x + 1}$ **and** $\mathbf{y = -x^2 = 4}$.

The Hyperbola

If a right circular cone is intersected by a plane parallel to its axis, part of a hyperbola is formed.

The equation of a hyperbola at the origin and with foci on the x-axis is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

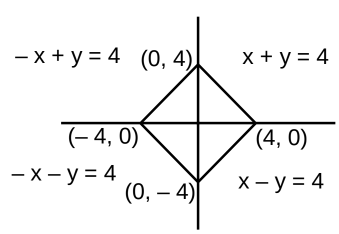
Ex.10 Find the area enclosed by the figure | x | + | y | = 4.

Sol. The four possible lines are

$x + y = 4$; $x - y = 4$; $-x - y = 4$ and $-x + y = 4$.

The four lines can be represented on the coordinates axes as shown in the figure. Thus a square is formed with the vertices as shown. The side of the square is $\sqrt{[(0-4)^2 + (-4-0)^2]} = 4\sqrt{2}$.

The area of the square is $(4\sqrt{2})^2 = 32$ sq. units.



Ex.11 If point (t, 1) lies inside circle $x^2 + y^2 = 10$, then t must lie between

Sol. As (t, 1) lies inside circle, so its distance from centre i.e. origin should be less than radius i.e. $\sqrt{10}$.

$\therefore$ Distance from origin = $t^2 + 1 \le 10 \Rightarrow t^2 - 9 \le 0 \Rightarrow -3 \le t \le 3$.

Ex.12 Find the equation of line passing through (2, 4) and through the intersection of line $4x - 3y - 21 = 0$ and $3x - y - 12 = 0$?

Sol. $4x - 3y - 21 = 0$ .....(1)

$3x - y - 12 = 0$ ....(2)

Solving (1) and (2), we get point of intersection as x = 3, y = – 3.

Now we have two points (3, –3) & (2, 4)

Slope of line m = $\frac{4+3}{2-3} = -7$

So equation of line $y + 3 = -7(x - 3)$ or $7x + y - 18 = 0$.

<u>Alternate method:</u>

Equation of line through intersection of $4x - 3y - 21 = 0$ and $3x - y - 12 = 0$ is

$(4x - 3y - 21) + k(3x - y - 12) = 0$.

As this line passes through (2, 4)

$(4 \times 2 - 3 \times 4 - 21) + k(3 \times 2 - 4 - 12) = 0$ or $k = -\frac{5}{2}$

So, equation of line is

$(4x - 3y - 21) - \frac{5}{2}(3x - y - 12) = 0$. or $7x + y - 18 = 0$.