Geometry – 2

(Polygons & Circle)

Polygons

A closed plane figure made up of several line segments that are joined together is called a polygon. The sides do not cross each other. Exactly two sides meet at every vertex.

Types of Polygons:

Regular: all angles are equal and all sides are of same length. Regular polygons are both equiangular and equilateral.

Equiangular: all angles are equal.

Equilateral: all sides are of same length.

Properties of Polygon

Property 1	Each exterior angle of an **n** sided regular polygon is $\frac{360^o}{n}$ degrees
Property 2	Each interior angle of an **n**-sided equiangular polygon is $\frac{(n-2)\times 180^o}{n}$ degrees. Also as each pair of interior angle & exterior angle is linear, Each interior angle = 180^o – exterior angle.
Property 3	**Area** of a regular polygon = (1/2) N sin(360°/N) S^2 (N = No. of sides and S = length from center to a corner) The areas of some well known regular polygons are: (1) Hexagon = $6 \times \frac{\sqrt{3}}{4} a^2$ (2) Octagon = $2a^2 (1 + \sqrt{2})$ where 'a' is the length of a side
Property 4	Sum of all the interior angles of **n** sided polygon is $(n-2)\times 180^o$

Quadrilaterals

A polygon with 4 sides, is a quadrilateral

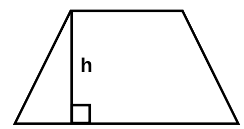
1. In a quadrilateral, sum of the four interior angles is equal to 360° and also the sum of exterior angles equal to 360°.
2. On the properties, quadrilaterals have been named differently, as given below.

Trapezium

(a) When one pair of opposite sides is parallel and no condition for other pair of opposite sides, then the quadrilateral is Trapezium.

(b) When the non – parallel sides are equal in length then the trapezium formed is "isosceles trapezium".

(c) The diagonals of isosceles trapezium are equal in length but do not bisect.



Parallelogram

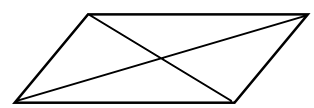
(a) Opposite sides are equals and parallel.

(b) Opposite angles are equal.

(c) Diagonals bisect each other

(d) Each diagonal divides the parallelogram into two triangles of equal area.

(e) Sum of any two adjacent angles is 180°.



Rhombus

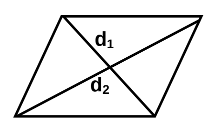
(a) All sides are equal, opposite sides are parallel.

(b) Opposite angles are equal.

(c) Diagonals bisect each other at right-angled. But they are not equal.

(d) Each diagonal divides the rhombus into two triangles of equal area.

(e) Area = $\frac{1}{2} \times d_1 \times d_2$



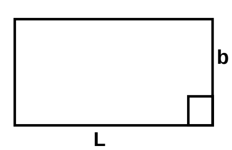
Rectangle

(a) Pair of opposite sides equal.

(b) Each angle is 90°.

(c) Diagonals are equal and bisect each other (not at right angles).

(d) Length of diagonal = $\sqrt{\ell^2 + b^2}$

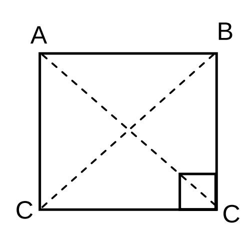


Square

(a) All sides are equal and angles equal.

(b) Diagonals equal and bisect at 90°.

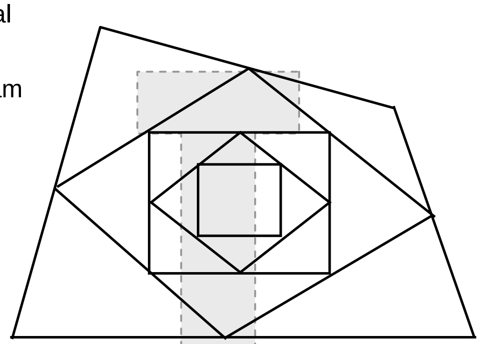
(c) When it is inscribed in a circle, the diagonal of square is equal to the diameter of the circle. But when circle is inscribed in a square, the side of the square is equal to diameter of the circle.



Important Results

If we join the mid-point of a quadrilateral we get a parallelogram and the mid-point of parallelogram will give a rectangle. If we again in join the mid-point of rectangle we get a rhombus and the mid point of rhombus will give you a square.

Quadrilateral
↓
Parallelogram
↓
Rectangle
↓
Rhombus
↓
Square

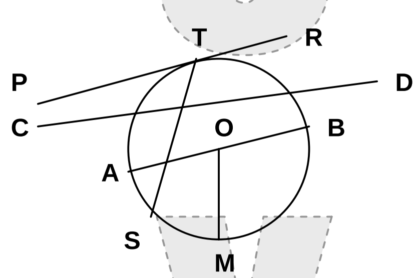


Circles

If O is a fixed point in a given plane, the set of points in the plane which are at equal distances from O will form a circle.

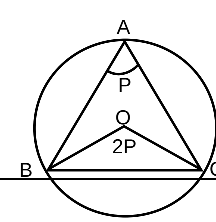
Parts of circle

In the figure below O is the centre of the circle of radius OM and diameter AB. Here diameter is always twice the radius of the circle. CD is the secant to the circle and cut the circle at two different points. The tangent PTR touches the circle at one and only one point. Also we have infinite number of tangent on a circle but at a point there will be one and only one tangent that can be drawn. The chord TS in a circle is the line which touches the circle at two different points and diameter is the longest chord.



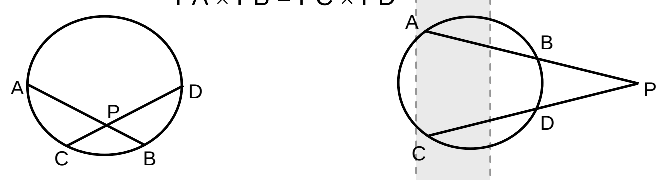
Properties of a Circle

1. If two chords of a circle are equal, their corresponding arcs have equal measure.
2. Measurement of an arc is the angle subtended at the centre. Equal arcs subtend equal angles at the center.
3. A line from centre and perpendicular to a chord bisects the chord.
4. Equal chords of a circle are equidistant from the centre.
5. When two circles touch, their centres and their point of contact are collinear.
6. If the two circles touch externally, the distance between their centres is equal to sum of their radii.
7. If the two circles touch internally, the distance between the centres is equal to difference of their radii.

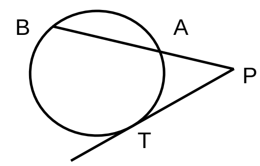


8. Angle at the centre made by an arc is equal to twice the angle made by the arc at any point on the remaining part of the circumference.
   Let O be the centre of the circle.
   $\angle BOC = 2\angle P$, when $\angle BAC = \angle P$
9. If two chords are equal then the arc containing the chords will also be equal.
10. There can be one and only one circle that touches three non-collinear points.
11. The angle inscribed in a semicircle is 90^o.
12. If two chords AB and CD intersect externally or internally at P, then

$$PA \times PB = PC \times PD$$

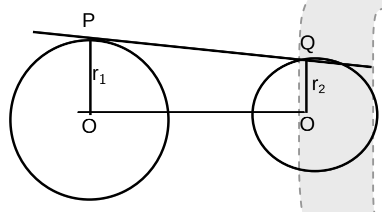


14. If PAB is a secant and PT is a tangent, then
    $PT^2 = PA \times PB$



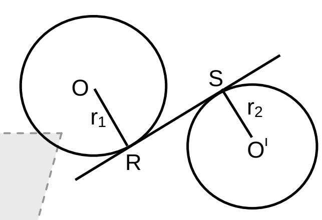
15. The length of the direct common tangent (PQ)

$$= \sqrt{(\text{The distance between their centres})^2 - (r_1 - r_2)^2}$$



16. The length of the transverse common tangent (RS)

$$= \sqrt{(\textit{The distance between their centres})^2 - (r_1 + r_2)^2}$$



Cyclic Quadrilateral

If a quadrilateral is inscribed in a circle i.e. all the vertex lies on the circumference of the circle, it is said to be cyclic quadrilateral.

1. In a cyclic quadrilateral, opposite angles are supplementary.
2. In a cyclic quadrilateral, if any one side is extended, the exterior angle so formed is equal to the interior opposite angle.

Alternate angle theorem

Angles in the alternate segments are equal.

In the given figure, AC is a Chord touching the circle at points A and C. At point A we have a tangent PAT making $\angle CAT$ and $\angle CAP$ with the chord AC. In the circle $\angle ABC$ and $\angle ADC$ are two angles in two different segments.

Here for ∠CAT, the ∠ADC is in alternate segment and for ∠CAP; the ∠ABC is in alternate segment. So according to the statement of the theorem the pair of these alternate angles are equal to each other.

Then, ∠CAT = ∠ ADC &

∠ PAC = ∠ ABC