Highest Common Factor

Every number has some factors but if two or more numbers taken together can have one or more common factors. Out of those common factors the greatest among them will be the highest common divisor or highest common factor of those numbers. Such as 12 and 18 have 1, 2, 3 and 6 as common factors but among them 6 is the highest common factor. So H.C.F. of 12 and 18 is 6.

Least Common Multiple

When we write the multiples of any two or more numbers taken together, we find that they have some common multiples. Out of those common multiples the smallest among them will be the least common multiple of those numbers. Such as 12 and 18 have 36, 72, 108, 144….. as common multiples but 36 is the least among them. So L.C.M. of 12 and 18 is 36.

Methods of finding L.C.M

(a) **Factorisation Method:** Resolve each one of the given numbers into prime factors. Then the product of the highest power of all the factors gives the L.C.M.

Methods of finding H.C.F

(a) **Factorisation method:**
Express each number as the product of primes and take the product of the least powers of **common** factors to get the H.C.F.

(b) **Division Method:**
Divide the larger number by smaller one. Now divide the divisor by the remainder. Repeat the process of dividing the preceding divisor by the remainder last obtained, till the remainder zero is obtained. The last divisor is the required H.C.F.

Ex.1 **Find the L.C.M of 2, 4, 6, 8, 10.**
Sol. **Write the numbers as the product of primes.**
2, 2^2, 2×3, 2^3, 2×5
Take the highest powers of all the primes. i.e., $2^3 \times 3 \times 5 = 120$.

Ex.2 **Find the H.C.F. of 2, 4, 6, 8, 10 using factorization method.**
Sol. For H.C.F, take the common prime factors. i.e., 2. (This is only the prime factor common in all the numbers).

Funda

i) Product of two numbers = L.C.M. × H.C.F.

ii) Product of n numbers = L.C.M of n numbers × Product of the HCF of each possible pair

Or

If the HCF of all the possible pairs taken is same then we will have
Product of n numbers = L.C.M of n numbers × (H.C.F of each pair)$^{(n-1)}$

iii) **If ratio of numbers is a : b and H is the HCF of the numbers Then**

LCM of the numbers = H × a × b = HCF × Product of the ratios.

vi) H.C.F of fractions = $\frac{\text{H.C.F. of numerators}}{\text{L.C.M. of denominators}}$.

vii) L.C.M. of fractions = $\frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$.

viii) If HCF (a, b) = H_1 and HCF (c, d) = H_2, then HCF (a, b, c, d) = HCF (H_1, H_2).

TIP

1. LCM is always a multiple of HCF of the numbers.
2. The numbers can be written as the multiple of HCF of them.

Important Results

IMPORTANT RESULTS

S.No.	Type of Problem	Approach of Problem
1.	Find the GREATEST NUMBER that will *exactly* divide x, y, z.	Required number = H.C.F. of x, y, and z (greatest divisor).
2.	Find the GREATEST NUMBER that will divide x, y and z leaving remainders a, b and c respectively.	Required number (greatest divisor) = H.C.F. of (x – a), (y – b) and (z – c).
3.	Find the LEAST NUMBER which is *exactly* divisible by x, y and z.	Required number = L.C.M. of x, y and z (least divided).
4.	Find the LEAST NUMBER which when divided by x, y and z leaves the remainders a, b and c respectively.	Then, it is always observed that (x – a) = (z – b) = (z – c) = K (say). ∴ Required number = (L.C.M. of x, y and z) – K.
5.	Find the LEAST NUMBER which when divided by x, y and z leaves the same remainder 'r' each case.	Required number = (L.C.M. of x, y and z) + r.
6.	Find the GREATEST NUMBER that will divide x, y and z leaving the same remainder in each case.	Required number = H.C.F of (x – y), (y – z) and (z – x).

Ex.3 What is the greatest number which exactly divides 110, 154 and 242?

Sol. The required number is the HCF of 110, 154 & 242.

110 = 2 × 5 × 11

154 = 2 × 7 × 11

242 = 2 × 11 × 11

∴ HCF = 2 × 11 = **22**

Ex.4 What is the greatest number, which when divides 3 consecutive odd numbers produces a remainder of 1.

Sol. If x, y, z be 3 consecutive odd numbers, then the required number will be the HCF of $x - 1$, $y - 1$ and $z - 1$.

Since x-1, y-1 & z-1 are 3 consecutive even integers, their HCF will be 2. So answer is **2.**

Ex.5 What is the highest 3 digit number, which is exactly divisible by 3, 5, 6 and 7?

Sol. The least no. which is exactly divisible by 3, 5, 6, & 7 is LCM (3, 5, 6, 7) = 210. So, all the multiples of 210 will be exactly divisible by 3, 5, 6 and 7. So, such greatest 3 digit number is **840. (210 $\times$ 4).**

Ex.6 In a farewell party, some students are giving pose for photograph, If the students stand at 4 students per row, 2 students will be left if they stand 5 per row, 3 will be left and if they stand 6 per row 4 will be left. If the total number of students are greater than 100 and less than 150, how many students are there?

Sol. If 'N' is the number of students, it is clear from the question that if N is divided by 4, 5, and 6, it produces a remainders of 2, 3, & 4 respectively. Since $(4 - 2) = (5 - 3) = (6 - 4) = 2$, the least possible value of N is LCM (4, 5, 6) – 2 = 60 – 2, = 58.

But, $100 < N < 150$. So, the next possible value is 58 + 60 = **118.**

Ex.7 There are some students in the class. Mr.X brought 130 chocolates and distributed to the students equally, then he was left with some chocolates. Mr Y brought 170 chocolates and distributed equally to the students. He was also left with the same no of chocolates as MrX was left. Mr Z brought 250 chocolates, did the same thing and left with the same no of chocolates. What is the max possible no of students that were in the class?

Sol. The question can be stated as, what is the highest number, which divides 130, 170 and 250 gives the same remainder, i.e. HCF $\left((170-130),(250-170),(250-130)\right)$.

I.e. HCF (40, 80, 120) **=** 40**.**