Introduction

Numbers form an integral part of our lives. In this lesson we will learn about the different types of numbers and the different categories under which they fall. The concepts discussed in this lecture will be your first step towards a general understanding of the mathematics requirements to clear MBA entrance exams. As we proceed with this lecture, you will realise that you have already learnt many of the concepts, included in this lesson, in school. This would further help build confidence in you. Although Number theory is important in the context of all the MBA entrance exams, it gains all the more importance for the students aiming for success in the CAT,

Understanding Numbers

A measurement carried out, of any quantity, leads to a meaningful value called the **Number**. This value may be positive or negative depending on the direction of the measurement and can be represented on the number line.

Natural Numbers (N)

The numbers 1, 2, 3, 4, 5…are known as natural numbers. The set of natural numbers is denoted by N. Hence, N = {1, 2, 3, 4…}. The natural numbers are further divided as even, odd, prime etc.

Whole Numbers (W)

All natural numbers together with '0' are collectively called whole numbers. The set of whole numbers is denoted by W, and W = {0, 1, 2, 3, ……}

Integers (Z)

The set including all whole numbers and their negatives is called a set of integers. It is denoted by Z, and Z = {– ∞, … – 3, – 2, – 1, 0, 1, 2, 3, ……. ∞}. They are further classified into Negative integers, Neutral integers and positive integers.

Negative integers (Z^-)

All integers lesser than Zero are called negative integers.

$Z^- = \{-1, -2, -3 \ldots -\infty\}$

Neutral integers (Z^0)

Zero is the only integer which is neither negative nor positive and it is called a neutral integer.

Positive integers (Z^+)

All integers greater than Zero are called positive integers.

$Z^+ = \{1, 2, 3, \ldots\ldots, \infty\}$

Classification of Numbers

i. Even Numbers:

All numbers divisible by 2 are called even numbers. E.g., 2, 4, 6, 8, 10 …Even numbers can be expressed in the form 2n, where n is an integer. Thus 0, – 2, – 6, etc. are also even numbers.

ii. Odd Numbers:

All numbers not divisible by 2 are called odd numbers. e.g. 1, 3, 5, 7, 9…Odd numbers can be expressed in the form (2n + 1) where n is any integer. Thus – 1, – 3, – 9 etc. are all odd numbers.

TIP

If a number has no prime factor equal to or less than its square root, then the number is a prime number.

iii. Prime Numbers:

A natural number that has no other factors besides itself and unity is a prime number.

Examples: 2, 3, 5, 7, 11, 13, 17, 19 …

Must Know

1 to 25 ⇒ 9 prime

1 t0 50 ⇒ 15 prime

1 to 100 ⇒ 25 prime

1 to 200 ⇒ 45 prime

Important Observation about prime numbers:

A prime number greater than 3, when divided by 6 leaves either 1 or 5 as the remainder. Hence, a prime number can be expressed in the form of 6K $\pm$ 1. But the converse of this observation is not true, that a number leaving a remainder of 1 or 5 when divided by 6 is not necessarily a prime number. For eg: 25, 35 etc

Ex.1 ***If a, a + 2, a + 4 are consecutive prime numbers. Then how many solutions 'a' can have?***

(1) one (2) two (3) three (4) more than three

Sol. No even value of 'a' satisfies this. So 'a' should be odd. But out of three consecutive odd numbers, atleast one number is a multiple of 3.

So, only possibility is a = 3 and the numbers are 3, 5, 7. **Answer: (1)**

iv. Composite Numbers: A composite number has other factors besides itself and unity. e.g. 8, 72, 39, etc. On the basis of this fact that a number with more than two factors is a composite we have only 34 composite from 1 to 50 and 40 composite from 51 to 100.

TIP

1 is neither a prime number nor a composite number

v. Perfect Numbers: A number is said to be a perfect number if the sum of ALL its factors excluding itself (but including 1) is equal to the number itself.

Or

The sum of all the possible factors of the number is equal to twice the number.

FUNDA: If the factors of any perfect number other than 1 are written and reciprocal of them are added together then result is always unity.

Example:

6 is a perfect number because the factors of 6, i.e., 1, 2 and 3 add up to the number 6 itself.

Also $\frac{1}{6}+\frac{1}{3}+\frac{1}{2} = \frac{1+2+3}{6} = \frac{6}{6}$ = 1(Unity)

Other examples of perfect numbers are 28, 496, 8128, etc. There are 27 perfect numbers discovered so far.

vi. Co-Prime numbers:

Two numbers are (prime or composite) said to be co-prime to one another, if they do not have any common factor other than 1. e.g. 35 & 12, since they both don't have a common factor among them other than 1.

TIP

The HCF of co-prime numbers is always 1

vii. Fractions

A fraction denotes part or parts of a unit. Several types are:

1. **Common Fraction:** Fractions whose denominator is not 10 or a multiple of it. e.g. $\frac{2}{3}$, $\frac{17}{18}$ etc.
2. **Decimal Fraction:** Fractions whose denominator is 10 or a multiple of 10.
3. **Proper Fraction:** In this the numerator < denominator e.g. $\frac{2}{10}$, $\frac{6}{7}$, $\frac{8}{9}$ etc. Hence its value < 1.
4. **Improper Fraction:** In these the numerator > denominator e.g. $\frac{10}{2}$, $\frac{7}{6}$, $\frac{8}{7}$ etc. Hence its value > 1.
5. **Mixed Fractions:** When a improper fraction is written as a whole number and proper fraction it is called mixed fraction. e.g. $\frac{7}{3}$ can be written as $2 + \frac{1}{3} = 2\frac{1}{3}$

Rational Numbers

Rational Number is defined as the ratio of two integers i.e. a number that can be represented by a fraction of the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$.They also can be defined as the non-terminating recurring decimal numbers. Such as 3.3333…., 16.123123….. are all rational numbers as they can be expressed in the form $\frac{p}{q}$.

Examples: Finite decimal numbers, whole numbers, integers, fractions i.e.

$\frac{3}{5}$, $\frac{16}{9}$, 0.666…∞, $\frac{2}{3}$, 7, 0 etc.

Irrational Numbers

Any number which can not be represented in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$ is an irrational number. On the basis of non-terminating decimals, irrational numbers are non-terminating non recurring decimals. Such as 3.4324546345…… is a non-terminating, non-repeating number.

Examples: π, $\sqrt{5}$, $\sqrt{7}$, e

Non–Terminating Decimal Numbers

When we divide any number by other number, either we get a terminating number or a non – terminating number. A non – terminating number on the basis of occurrence of digits after decimal can classified as following.

1. Pure Recurring Decimals:

A decimal in which all the figures after the decimal point repeat, is called a *pure recurring decimal*.

Examples: $0.\dot{6}$, $0.\overline{6}$ are examples of pure recurring decimals. ($0.\dot{6} = 0.\overline{6} = 0.666$ ……….)

2. Mixed Recurring Decimals: A decimal in which some figures do not repeat and some of them are repeated is called a ***mixed recurring decimal*.**

Examples: $0.2\overline{3}$, $0.35\overline{27}$ (($0.2\overline{3} = 0.23333$….., $0.35\overline{27} = 0.35272727$………)

3. Non – Recurring Decimals: A decimal number in which the figure don't repeat themselves in any pattern are called non-terminating non-recurring decimals and are termed as irrational numbers.

Converting Recurring Decimal as Fraction

All recurring decimals can be converted into fractions. Some of the common types can be 0.33….. , 0.1232323…, 5.33…., 14.23636363…. etc.

Pure Recurring to Fractions

FUNDA 1: If a number is of the form of 0.ababab……. then divide the repeating digits with as many 9's as we have repeated digits.

eg. $0.363636\ldots = \frac{36}{99} = \frac{4}{11}$

Mixed Recurring to Fractions

FUNDA 2: If N = 0.abcbcbc…. Then

$$N = \frac{abc - a}{990} = \frac{\text{Repeated \& non repeated digits} - \text{Non repeated digits}}{\text{As many 9's as repeated digits followed by as many zero as non-repeated digits}}$$

eg. $0.25757\ldots = \frac{257 - 2}{990} = \frac{255}{990} = \frac{17}{66}$

FUNDA 3: If N = a.bcbc…. Then

Write N = a + 0.bcbc….

Proceed as Funda 1

$5.3636\ldots = 5 + 0.3636\ldots = 5 + \frac{36}{99} = \frac{59}{11}$

Divisibility Test

No.	Rule	Example/s
2	Last digit of number is even or zero.	12**8**, 14**6**, 3**4** etc.
3	Sum of digits of a given number is divisible by 3	102, 192, 99 etc
4	Number formed by last two right hand digits of given number is divisible by 4 or 00	5**76**, 1**44** etc.
5	Last digit is either five or zero	11115**5**, 397**0**, 14**5** etc.
6	Number is divisible by both 2 and 3	714, 509796, 1728 etc.
7	The rule which holds good for numbers with more than 3 digits is as follows: (A) Group the numbers in three from units digit (B) Add the odd groups and even groups separately (C) The difference of the odd and even groups should be 0 or divisible by 7	Let's take 85437954 . The groups are 85, 437, 954 Sum of odd groups = 954 + 85 = 1039 Sum of even groups = 437 Difference = 602. Which is divisible by 7, hence the number is divisible by 7.

8	Number formed by the last three right hand digits of a number is divisible by '8' or 000.	512, 4096, 1304 etc.
9	Sum of its digits is divisible by 9.	1287, 11583, 2304 etc.
10	Unit digit is zero.	100, 170, 10590 etc.
11	When the difference between the sums of digits in the odd and even places taken from right to left is either zero or a multiple of 11.	e.g.17259 Sum of digits in even places = 7 + 5 = 12, Sum of digits in the odd places = 1 + 2 + 9 = 12 Hence 12 – 12 = 0.
12	It is divisible by 3 & 4 both.	672, 8064 etc.
13	The rule which holds good for numbers with more than 3 digits is as follows: (A) Group the numbers in three from units digit (B) Add the odd groups and even groups separately (C) The difference of the odd and even groups should be 0 or divisible by 13	Let's take 35250799415 The groups are 035, 250, 799, 415 Sum of odd groups = 035 + 799 = 834 Sum of even groups = 250 + 415 = 665 Difference = 834 – 615 =169 which is divisible by 13, hence the number is divisible by 13.

FUNDA:

How to calculate remainder, when a number is divided by 11, without division?

Step 1: Add all the odd place numbers **(O)** and even place numbers **(E)** counted from right to left.

Step 2: If **O – E** is positive, remainder will be the difference less than 11.

Step 3: If **O – E** is negative, remainder should be (11 – difference).

Ex.2 ***What is the remainder when 2354789341 is divided by 11?***

Sol. Odd place digit sum (O) = 1 + 3 + 8 + 4 + 3 = 19.

Even place digits sum (E) = 4 + 9 + 7 + 5 + 2 = 27.

Difference (D) = 19 – 27 = – 8

Remainder = 11 – 8 = **3.**

TIP

When any number with even number of digits is added to its reverse, the sum is always divisible by 11. **e.g.** 2341 + 1432 = 3773, which is divisible by 11.

Any number written 6 times consecutively will be divisible by 7 and 13

Ex.3 If 567P55Q is divisible by 88; Find the value of P + Q.

(1) 11 (2) 12 (3) 5 (4) 6 (5) 10

Sol. The number is divisible by 8 means; the number formed by the last 3 digits should be divisible by 8 which are 55Q. Only Q = 2 satisfy this. From the divisibility rule of 11, (2 + 5 + 7 + 5) – (5 + P + 6) is divisible by 11. So 8-P is divisible by 11. if P= 8, then only it is possible. So P = 8 and Q = 2.
So **P + Q = 10. Answer: (5)**

Ex.4 If the first 100 natural numbers are written side by side to form a big number and it is divided by 8. What will be the remainder?

(1) 1 (2) 2 (3) 4
(4) 7 (5) cannot be determined

Sol. The number is 1234…..9899100
According to the divisibility rule of 8, we will check only the last 3 digits.
If 100 is divided by 8, the remainder is 4. **Answer: (3)**

Ex.5 What will be the remainder when 4444……..44 times is divided by 7?

(1) 1 (2) 2 (3) 5 (4) 6 (5) 0

Sol. If 4 is divided by 7, the remainder is 4.
If 44 is divided by 7, the remainder is 2.
If 444 is divided by 7, the remainder is 3
By checking like this, we come to know that 444444 is exactly divisible by 7.
So if we take six 4's, it is exactly divisible by 7. Similarly twelve 4's is also exactly divisible by 4 and 42 4's will be exactly divisible by 7. So out of 44, the remaining two 4,s will give a remainder of 2.
So, answer (2).

Absolute value of a number

The modulus of a number is the absolute value of the number or we can say the distance of the number from the origin. The absolute value of a number a is defined as

$|a| = a$, if $a \geq 0$
$= -a$, if $a \leq 0$

> $|a|$ is read as MODULUS of a or **Mod a**

Example: $|79| = 79$ & $|-45| = -(-45) = 45$
Also, $|x - 3| = x - 3$, if $x \geq 3$
$= 3 - x$, if $x < 3$.

Always Keep in Mind

The number 1 is neither prime nor composite.

1) 2 is the only even number which is prime.
2) $(x^n + y^n)$ is divisible by $(x + y)$, when n is an odd number.
3) $(x^n - y^n)$ is divisible by $(x + y)$, when n is an even number.
4) $(x^n - y^n)$ is divisible by $(x - y)$, when n is an odd or an even number.
5) The difference between 2 numbers (xy) – (yx) will always be divisible by 9.
6) The square of an odd number when divided by 8 will always give 1 as a remainder.

7) Every square number is a multiple of 3 or exceeds a multiple of 3 by unity.
8) Every square number is a multiple of 4 or exceeds a multiple of 4 by unity.
9) If a square number ends in 9, the preceding digit is even.
10) If m and n are two integers, then (m + n)! is divisible by m! n!
11) $(a)^n / (a + 1)$ leaves a remainder of $\begin{cases} a & \text{if } n \text{ is odd} \\ 1 & \text{if } n \text{ is even} \end{cases}$.
12) Product of n consecutive numbers is always divisible by n!.

Ex.6 If 'X' is an even number; Y is an odd number, then which of the following is even?

(1) $X^2 + Y$ (2) $X + Y^2$ (3) $X^2 + Y^2$ (4) X^2Y^2 (5) None of these

Sol. Since X is even, X^2 is even.
Y is odd, Y^2 is odd
So options (1), (2), (3) are even + odd = odd.
Option (4) is (even) (odd) = Even. **Answer: (4)**

Ex.7 What is the difference between 0.343434......and 0.2343434…… in fraction form?

(1) $\frac{6}{55}$ **(2)** $\frac{6}{11}$ **(3)** $\frac{9}{55}$ **(4)** $\frac{9}{13}$ **(5)** $\frac{5}{11}$

Sol. 0.343434….. = $\frac{34}{99}$ and 0.23434….. = $\frac{234-2}{990} = \frac{232}{990}$

$\therefore$ Difference = $\frac{34}{99} - \frac{232}{990} = \frac{108}{990} = \frac{6}{55}$

Ex.8 How many of the following numbers are divisible by at least 3 distinct prime numbers 231, 750, 288 and 1372?

(1) 0 (2) 1 (3) 2 (4) 3 (5) 4

Sol. $231 = 3 \times 7 \times 11$ (3 prime factors)
$750 = 2 \times 3 \times 5^3$ (3 prime factors)
$288 = 2^5 \times 3^2$ (only 2 prime factors)
$1372 = 2^2 \times 7^3$ (only 2 prime factors)
So, only 231 & 750 has 3 prime factors. **Answer: (3)**

Ex.9 $n^3 + 6n^2 + 11n + 6$ (where n is a whole no) is always divisible by

(1) 4 (2) 5 (3) 6 (4) 8 (5) 12

Sol. $n^3 + 6n^2 + 11n + 6 = (n + 1)(n + 2)(n + 3)$.
Product of 3 consecutive numbers is always divisible by **3! = 6.**
(or)
Take n = 0, 1, 2, 3 and check it is always divisible by **6.** **Answer: (3)**

Ex.10 What is the remainder, if 351 × 352 × 353 × - - - - - - - × 356 is divided by 360?

(1) 0 (2) 1 (3) 2 (4) 3 (5) 359

Sol. Since the given is the product of 6 consecutive numbers, it is always divisible by 6! = 720.
$\Rightarrow$ it is divisible by 360 also. So, the remainder will be **0.** **Answer: (1)**

Ex.11 If a is an even integer, except 0, b is a positive integer c is an odd integer, then $(ab)^c$ is always

(1) odd **(2) Positive** **(3) Negative**

(4) Even **(5) cannot be determined**

Sol. Here we don't know whether 'a' is negative or positive. Still we can say that $(ab)^c$ is always even because 'a' is even & hence ab is even and any exponent of an even number is always even.

Answer: (4)

Ex.12 The digits of a 3 digit no reversed to form another number. The difference between this no and the original number is always divisible by.

(1) 2 **(2) 4** **(3) 6** **(4) 8** **(5) 11**

Sol. If abc is the number

then cba is the other number.

abc = 100a + 10b + c and

cba = 100c + 10b + a.

Difference = 99(a ~ c).

This is always divisible by 9 & 11. **Answer: (5)**

Ex.13 How many two digit numbers can be formed such that sum of its digits is equal to the product of the digits?

Sol. If xy is the number, then

$x + y = xy \Rightarrow x(y-1) - y = 0 \Rightarrow (x-1)(y-1) = 1$

Only (0, 0) & (2, 2) satisfy this

But 00 is not a 2 digit no.

So only 22 satisfy this.

Only 1 such number satisfies.

Ex.14 The sum of all the two digit numbers which has both the digits even.

(1) 2060 **(2) 1080** **(3) 1272** **(4) 2160** **(5) 1560**

Sol. The no's are 20, 22, - - - - , 28 ← 120

40, 42, - - - - , 48 ← $120 + 20 \times 5 = 220$

60, 62, - - - - , 68 ← 320

80, 82, - - - - , 88 ← 420

Sum = 120 + 220 + 320 + 420 = **1080** **Answer: (2)**

Ex.15 If we add a two digit number to another two digit number (N), the digits of N will get reversed. Again if we add the same no to this result, again the same digits of N will come, but with a zero in between them. What is the number added.

(1) 27 **(2) 36** **(3) 45** **(4) 54** **(5) 50**

Sol. Let xy be the number (N).

N = xy = 10x + y. ……..(1)

If we add a two digit number A (say), it will become yx = 10y + x = A + N ……..(2)

Again, if we add the same number, it will be x0y = 100x + y = 2A + N ……..(3)

(2) – (1) = (3) – (2) = A

$\therefore (10y + x) - (10x + y) = (100x + y) - (10y + x) \Rightarrow y = 6x$

Since x and y all digits, only x = 1 and y = 6 will satisfy this.

$\therefore A = (2) - (1) = 9y - 9x = 45$ **Answer: (3)**