Defining a Set

"Set" is synonymous with the words "collection", "aggregate", "class" and is comprised of elements/objects/members.

A **set** is a **well-defined collection** of **elements**. By well-defined elements it means that given a set and an element, it must be possible to decide whether or not the element belongs to the set. A set can be described in any one of the following ways. For example, the set of beautiful Actress of Bollywood or the set of Good Players of cricket in India are not sets as the world beautiful and Good are not well defined. What are the criteria of choosing an actress as beautiful and which player is said to be a good player of cricket. But on the other hand if we say Set of good player of cricket in India those has played at least 25 international games, will be a set as the word Good is now well defined.

TIP

If a is the element of the set A then we write $a \in A$ and read it as **"a belong to Set A"**

Representation of a Set

A set can be described in two different ways.

1. Roaster Form:

A **set** is described by listing elements, separated by commas, within brackets. For example, A = {a, e, i, o, u} is a set of vowels of English alphabets and a finite set and N = {2, 4, 6,...} is a set of even natural number and is a infinite set. Also, repetition of an element has no effect. For example {1, 2, 3, 2} is the same set as {1, 2, 3}.

FACT

If a set contain countable number of elements it is called FINITE set and if contain uncountable number of elements it is called INFINITE set.

2. Set Builder Form:

A set can also be described by a characterizing property P(x) of its elements x. In such a case the set is described by {x **|** P (x) holds} or {x : P(x) holds}, which is read as 'the set of all x such that P(x) holds'. For example,

A = {x **|** x is the root of $x^2 + 5x – 6 = 0$}

Cardinal number of a set

The number of elements contained in a set is known as the cardinal number of that set. On the basis of cardinal number a set can be an Empty Set or a Singleton Set. A set is said to be **empty** or **void** or **null set** if it has no elements and its cardinal number is 0. It is denoted by the symbol ϕ or { } such as Set of odd numbers divisible by 2. And a set is singleton set if the cardinal number is 1 i.e. it contains only one element, such as Set of lady Prime-minister of India.

If a set A has 5 elements then its cardinal number is written as n(A) = 5, and read as "Cardinal number of set A is 5".

TIP

Let A be a finite set having n elements. Then the total number of sub sets of A is 2^n and number of proper subsets of A is $2^n – 1$.

Subsets

Let A and B be two sets. If every element of A is contained in B, then A is called the subset of B.

If A is subset of B, we write $A \subset B$, which is read as "A is a subset of B" or "A is contained in B".

If A $\subset$ B then a $\in$ A $\Rightarrow$ a $\in$ B. (The symbol $\Rightarrow$ stands for "implies" and $\in$ stands for "belongs to").

Let P = {a, b, c}, then subsets of P are ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c} i.e. it has 8 number of subsets and Empty set is the subset of every. Also every set is the subset of itself and it is improper subset of the set. Every set has one improper subset and other subsets as proper subsets.

If A $\subset$ B and also B $\subset$ A, then we can say every element of A is present in B and every element of B is present in A. These types of sets having same number and identical elements are known as ***Equal sets.***

Power Set

The set of subsets of a set is the power set of the set. If A is a set then its Power set is denoted as P(A).

If A = {a, b, c}, then P(A) = {ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}

The cardinal number of any set A is 2^n, where n = number of elements present in set A.

Universal Set

In any discussion in theory, there happens to be a set U that contains all sets under consideration. Such a set is called the universal set. Thus a set that contains all sets in a given context is called the universal set. For example, in plane geometry the set of all points in the plane is the universal set.

Some Important Universal Sets:

N = Set of all natural numbers = {1, 2, 3, 4,…}

Z or **I** Set of all integers = {…– 3, – 2, – 1, 0, 1, 2, 3,..}

$\mathbf{Z^+}$ = Set of all positive integers = {1, 2, 3,…} = N

$\mathbf{Z^-}$ = Set of all negative integers = {– 1, – 2, – 3,..}

W = Set of all whole numbers = {0, 1, 2, 3,…}

$\mathbf{Z_0}$ = The set of all non-zero integers = {± 1, ± 2, ± 3,..}

Q = The set of all rational numbers.

$$= \left\{\frac{p}{q} : p, q \text{ are integers, } q \neq 0\right\}$$

R = The set of all real numbers.

R – Q = The set of all irrational numbers.

e.g. $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$,… π ,e, log 2 etc. are all irrational numbers.

Operations of sets

Union of Sets:

The union of two sets A and B, is a set containing all the elements present in set A or in set B. The union of set A and B is represented as A U B.

So, **A U B = {x | x $\in$ A or x $\in$ B}**

For example:

A = {5, 7, 8, 9, 11} and B = {car, house, ball, sofa), then

A U B = {5, 7, 8, 9, 11, car, house, ball, sofa}

TIP

(a) Two sets are disjoint sets if and only if they do not have any common element

(b) The cardinal number of their intersection is Zero.

Intersection of Sets:

The intersection of two sets A and B, is a set containing all the common elements present in set A or in set B. The intersection of set A and B is represented as A $\cap$ B.

So, **A $\cap$ B = {x | x $\in$ A and x $\in$ B}**

For example

A = {5, 7, 8, 9, 11} and B = {3, 4, 5, 6, 7, 8}, then

$A \cap B = \{5, 7, 8\}$

Difference of sets:

A – B is the difference of set A from set B and is defined as the set of elements present in set A but not in set B.

So, **$A - B = \{x \mid x \in A \text{ and } x \notin B\}$**

FACT

Difference of two sets is not commutative. i.e.

$A - B \neq B - A$

Complement of a set:

The complement of set A in U the set of those elements which are present in Universal set but not present in set A. Compliment of A is denoted by A' The shaded part represents A'.

So, **$A' = \{x \mid x \in U \text{ and } x \notin A\}$**

(a) Also the Compliment of set A is defined as the difference of Universal set U and set A. i.e.$\Rightarrow$ **$A' = U - A$**

Facts and Rules:

i. $A \cup A = A$ & $A \cap A = A$

ii. $A \cup \phi = A$ & $A \cap \phi = \phi$

iii. $A \cup U = U$ & $A \cap U = A$

iv. $A \cup B = B \cup A$ & $A \cap B = B \cap A$

v. $A \cup A' = U$ & $A \cap A' = \phi$

vi. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

vii. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - (C \cap A) + n(A \cap B \cap C)$

viii. $n(A - B) = n(A) - n(A \cap B)$

Ex.1 ***In a certain city only two newspapers A & B are published. It is known that 25% of the city population reads A & 20% read B while 8% read both A & B. It is also known that 30% of those who read A but not B, look into advertisements and 40% of those who read B but not A, look into advertisements while 50% of those who read both A & B look into advertisements. What % of the population reads an advertisement?***

Sol. Let A & B denote sets of people who read paper A & paper B respectively and in all there are 100 people, then n(A) = 25, n(B) = 20, $n(A \cap B) = 8$.

Hence the people who read paper A only i.e. $n(A - B) = n(A) - n(A \cap B) = 25 - 8 = 17$.

And the people who read paper B only i.e. $n(B - A) = n(B) - n(A \cap B) = 20 - 8 = 12$.

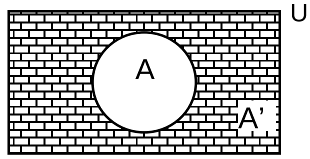
Now percentage of people reading an advertisement

= [(30% of 17) + (40% of + 12) + (50% of 8)]% = 13.9 %.

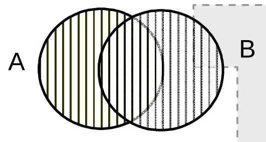
Venn Diagrams

Venn diagram is the pictorial representation of the set and also the operation involved in the sets. We often use circles to represent the sets and overlapping of the circles to represent the common elements in two or more sets. The universal set U is represented by interior of a rectangle and its subsets are represented by interior of circles within the rectangle. Below are some examples of Venn-diagrams.

1. U = universal set
   A = subset of U
   A' = complement of A

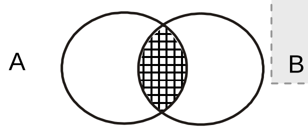


2. The representation of A U B in Venn diagram is



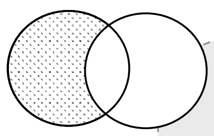
A U B

3. The representation of $A \cap B$ in Venn diagram is



$A \cap B$

4. Difference of set A from B i.e. A – B



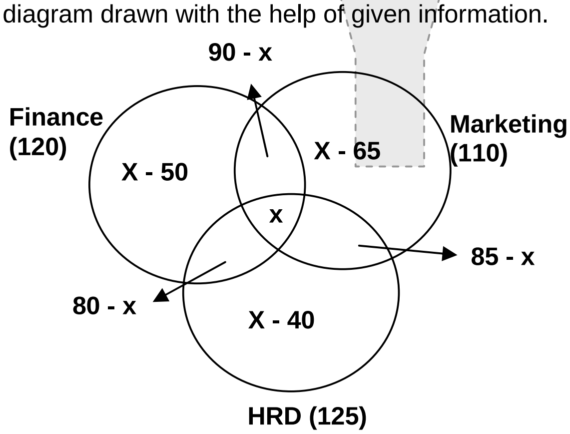
Maximum and Minimum elements in a set

Let us consider the formula,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ and imagine that the value of $n(A \cup B)$ is not given and asked to find the value of $n(A \cap B)$, then we will get the range of the values. Let us understand the concept of maximum and minimum values with the help of an example.

Ex.2 ***In a B – school there are three specializations in Management course and a student is free to specialize in any number of fields. These specializations are Finance, Marketing and HRD. If 120 students specialize in Finance, 110 in Marketing and 125 are in HRD. 90 students have finance and Marketing both as their specialization, 85 have marketing and HRD while 80 have both finance and HRD. What can be the minimum and maximum number of students who can specialization in all the three fields?***

Sol. Let us consider that the number of student who enroll in all the three streams be x. Thus according to the Venn diagram drawn with the help of given information.



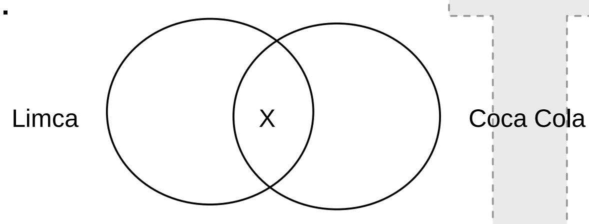
Now for the minimum value of x, we have $x - 65 \geq 0 \Rightarrow x \geq 65$

And for maximum value of x, we have $80 - x \geq 0 \Rightarrow x \leq 80$

Hence $65 \leq x \leq 80$. So we have the range of the values that can be the required solution.

Ex.3 ***If in a Survey, Organized by any N.G.O on the cold drinks after effects found that 80% of the total people like Coca - Cola and 70% like Limca. What can be the minimum and maximum number of people who drink both the drinks?***

Sol.



Here $n(L \cup C) \leq 100\%$

Using the formula, $n(L) + n(C) - n(L \cap C) = n(L \cup C) \leq 100\%$

$80 + 70 - X \leq 100$

Solving we get $X \geq 50\%$, Also $X \leq 70\%$

Minimum value = 50% and Maximum value = 70%.