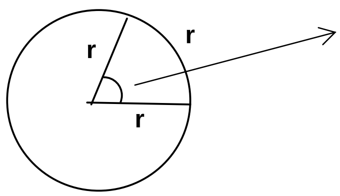
Angles and their relationship

Angles are measured in many units' viz. degrees, minutes, seconds, radians, gradients.

Where 1 degree = 60 minutes, 1 minute = 60 seconds,

$$\pi \textbf{ radians} = 180° = 200^g$$

$\Rightarrow$ 1 radian = 180°/π and 1 degree = π/ 180 radians.

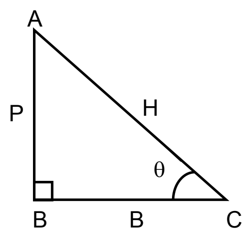


The angle at the centre is of 1 radian

> **Do you know?**
>
> 1 radian is the angle made at the centre by the arc of length equal to the radius of the circle.

Basic Trigonometric Ratios

In a right triangle ABC, if θ be the angle between AC & BC.



If θ is one of the angle other then right angle, then the side opposite to the angle is perpendicular (P) and the sides containing the angle are taken as Base (B) and the hypotenuse (H). In this type of triangles, we can have six types of ratios. These ratios are called trigonometric ratios.

$\text{Sin}\ \theta = \frac{P}{H}$, $\text{Cos}\ \theta = \frac{B}{H}$ $\text{Tan}\ \theta = \frac{P}{B}$

$\text{Cosec}\ \theta = \frac{H}{P}$, $\text{Sec}\ \theta = \frac{H}{B}$, $\text{Cot}\ \theta = \frac{B}{P}$

Important Formulae

For any angle θ:

1. $\sin^2\theta + \cos^2\theta = 1$ [Note $\sin^2\theta = (\sin\theta)^2$ and not $(\sin\theta^2)$]
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \text{cosec}^2\theta$

Range of Values of Ratios

If $0 \le \theta \le 360^o$, then the values for different trigonometric ratios will be as follows.

1. $-1 \le \text{Sin}\ \theta \le 1$
2. $-1 \le \cos\theta \le 1$
3. $-\infty \le \tan\theta \le \infty$
4. $-\infty \le \cot\theta \le \infty$
5. $-\infty \le \text{Sec}\ \theta \le -1$ & $1 \le \text{Sec}\ \theta \le \infty$
6. $-\infty \le \text{Cosec}\ \theta \le -1$ & $1 \le \text{Cosec}\ \theta \le \infty$

TIP

Maximum and Minimum values of sin θ or cos θ are + 1 and −1 respectively.

Sign of Trigonometric ratios

We divide the angle at a point (i.e. 360°) into 4 parts called quadrants. In the first quadrant all the trigonometric ratios are positive

II	I
90° → 180°	0° → 90°
180° → 270°	270° → 360°
III	IV

SOME MORE RESULTS:

Angles \ T. Ratios	Sin	Cos	Tan	Cot	Sec	Cosec
90 - θ	Cos θ	Sin θ	Cot θ	Tan θ	Cosec θ	Sec θ
90 +θ	Cos θ	– Sin θ	– Cot θ	– Tan θ	– Cosec θ	Sec θ
180 - θ	Sin θ	– Cos θ	– Tan θ	– Cot θ	– Secθ	Cosecθ
180 + θ	– Sin θ	– Cos θ	Tan θ	Cot θ	– Secθ	– Cosecθ
270 - θ	– Cos θ	– Sin θ	Cot θ	Tan θ	– Cosec θ	– Sec θ
270 + θ	– Cos θ	Sin θ	– Cot θ	– Tan θ	Cosec θ	– Sec θ
360 - θ	– Sin θ	Cos θ	– Tan θ	– Cot θ	Secθ	– Cosecθ
360 + θ	Sin θ	Cos θ	Tan θ	Cot θ	Secθ	Cosecθ

TIP

Add: **A**ll positive
Sugar: **S**in positive
To: **T**an positive
Coffee : **C**os positive

Ex.1 ***Simplify*** $\dfrac{\tan(90^o+\theta)\sin(180^o+\theta)\sec(270^o+\theta)}{\cos(270^o-\theta)\text{cos ec}(180^o-\theta)\cot(360^o-\theta)}$.

Sol. $\tan(90^o+\theta) = -\cot\theta$, $\sin(180^o+\theta) = -\sin\theta$

$\sec(270^o+\theta) = \text{cosec}\ \theta$, $\cos(270^o-\theta) = -\sin\theta$

$\text{cosec}(180^o-\theta) = \text{cosec}\ \theta$, $\cot(360^o-\theta) = -\cot\theta$

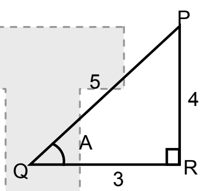
$\therefore$ Given expression = $\dfrac{(-\cot\theta)(-\sin\theta)(\text{cos ec}\,\theta)}{(-\sin\theta)(\text{cos ec}\,\theta)(-\cot\theta)} = 1$.

Ex.2 ***If cot A =*** $\mathbf{\frac{3}{4}}$ ***, find the value of 3 cos A + 4 sin A, where A is in the first quadrant.***

Sol. cot A = $\frac{3}{4}$

$\Rightarrow$ tan A = $\frac{4}{3}$

$\Rightarrow \frac{\text{Perpendicular}}{\text{base}} = \frac{4}{3}$



Draw a right ΔPQR in which $\angle Q$ = A,
PR = 4, QR = 3

$\Rightarrow PQ = \sqrt{(PR)^2 + (QR)^2}$

$= \sqrt{(4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25}$

$\Rightarrow$ PQ = 5 units

$\therefore$ cos A = $\frac{\text{base}}{\text{Hypotenuse}}$

cos A = $\frac{3}{5}$ and sin A = $\frac{PR}{PQ} = \frac{4}{5}$

3 cos A + 4 sin A = $3 \times \frac{3}{5} + 4 \times \frac{4}{5} = \frac{9}{5} + \frac{16}{5}$

3 cos A + 4 sin A = $\frac{25}{5} = 5$.

Ex.3 ***Find the value of*** $\left(\frac{\sin 35^\circ}{\cos 55^\circ}\right)^2 + \left(\frac{\cos 55^\circ}{\sin 35^\circ}\right)^2 - 2\cos 60^\circ$

Sol. $\left(\frac{\sin 35^\circ}{\cos 55^\circ}\right)^2 + \left(\frac{\cos 55^\circ}{\sin 35^\circ}\right)^2 - 2\cos 60^\circ$

$= \left[\frac{\sin(90^\circ - 55^\circ)}{\cos 55^\circ}\right]^2 + \left[\frac{\cos(90^\circ - 35^\circ)}{\sin 35}\right]^2 - 2 \times \frac{1}{2}$

$= \left(\frac{\cos 55^\circ}{\cos 55^\circ}\right)^2 + \left(\frac{\sin 35^\circ}{\sin 35^\circ}\right)^2 - 1$ $\left\{\begin{array}{l}\because \sin(90^o - \theta) = \cos\theta \\ \cos(90^o - \theta) = \sin\theta\end{array}\right\}$

= 1 + 1 – 1 = 1

Values of trigonometric Ratio for some special angles:

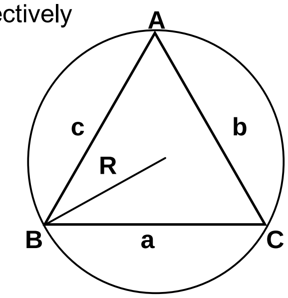
θ	0°	30°	45°	60°	90°
sin θ	0	1/2	1/√2	√3/2	1
cos θ	1	√3/2	1/√2	1/2	0
tan θ	0	1/√3	1	√3	∞ (Not defined)

Properties of triangle

Sine Rule

In any triangle ABC if AB, BC, AC be represented by c, a, b respectively

then we have $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$



Where R is circum – radius = $\frac{abc}{4 \times \text{Area of triangle}}$

Cosine Rule

In a triangle ABC of having sides of any size, we have the following rule;

Cos A = $\frac{b^2 + c^2 - a^2}{2bc}$ Cos B = $\frac{c^2 + a^2 - b^2}{2ac}$ Cos C = $\frac{a^2 + b^2 - c^2}{2ab}$

Area of triangle

Area Δ = $\frac{1}{2}$bc sin A = $\frac{1}{2}$ac sin B = $\frac{1}{2}$ab sin C

= $\sqrt{S(S-a)(S-b)(S-c)}$ where S = semi perimeter

Ex.4 ***Solve the equation*** $\sec^2 x - 2\tan x = 0$

Sol. $\sec^2 x - 2\tan x = 0$

$1 + \tan^2 x - 2\tan x = 0$ $\{\because \sec^2\theta - \tan^2\theta = 1\}$

$\tan^2 x - 2\tan x + 1 = 0$

$(\tan x - 1)^2 = 0$

$\tan x = 1$

$x = 45^o$.