

**VEDIC MATHEMATICS**

**ADDITION AND SUBTRACTION**

Use of following tips makes the subtraction and addition operation easy and less time consuming, in this method we change the numbers in the easy form and then we solve accordingly. Let's take an example to understand this method.

Suppose we have to add 689 and 95.

We know 95 is nearer to 100, by keeping this in mind we can add 100 to 689 and subtract 5 later.

Hence  $689 + 100 = 789 - 5 = 784$ , which is the required answer.

Take some other examples:

$67 + 693 = ?$

It can be solved like,  $67 + 700 = 767 - 7 = 760$ .

$454 + 27 = ?$

It can be solved like,  $454 + 30 = 484 - 3 = 481$ .

Similarly subtraction operation can be solved using these tips.

Let's take some examples:

1.  $367 - 37 = ?$

It can be solved like,  $367 - 40 = 327 + 3 = 330$ .

2.  $289 - 58 = ?$

It can be solved like,  $289 - 60 = 229 + 2 = 231$ .

**SUM OF SERIES**

Sometime we have to add many numbers which are in series, i.e. they are in certain fashion.

For example,

1. **Consecutive numbers:**

1, 2, 3, 4, 5 etc; or 12, 13, 14...

2. **Consecutive EVEN numbers:**

2, 4, 6, 8, etc; or 12, 14, 16...

3. **Consecutive ODD numbers:**

3, 5, 7, 9, etc; or 13, 15, 17,

4. **Consecutive negative numbers:**

- 1, - 2, - 3, etc; or - 11, - 12, - 13.....

5. **Consecutive (EVEN) negative numbers:**

- 2, - 4, - 6 etc; or - 12, - 14, - 16.....

6. **Consecutive (ODD) negative numbers:**

- 3, - 5, - 7, etc; or - 13, - 15, - 17...

For computing above type of sums we can use following formula.

$$S = (F + L) (N) / 2$$

Where:

S = Sum of all numbers

F = First number in sequence

L = Last number in sequence

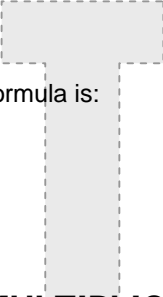
N = Number of Terms in sequence

Other examples in which, we can use this formula is:

1, 2, 3 ..... (Up to 50)

3, 7, 11, 15, 19, 23

4, 9, 14, 19, 24, 29



## MULTIPLICATION

### **METHODS FOR MULTIPLICATION OF NUMBER BY MULTIPLE OF 10 (i.e. BY 10, 100, 1000 etc.)**

This is quite simple just put the same number of zeroes behind the number as behind 1.

**Example:**  $23 \times 100$

Here there are two zeroes behind 1, hence by putting two zeroes behind 23. We will get the answer.

Therefore  $23 \times 100 = 2300$

Take another example e.g.  $45 \times 1000 = 45000$



### **METHODS FOR MULTIPLICATION OF NUMBER BY 5**

Let n is an number which is to be multiplied by 5.

i.e.  $n \times 5 = ?$

Now n could be either even or odd.

Now if n is even, just half the number and put zero behind that.

For example  $44 \times 5$

Here 44 is an even number, now half of the 44 is 22 and by putting '0' it become 220. Hence answer is 220.

Now if n is odd, subtract one from n, and half that number (i.e.  $n - 1$ ) and put five behind that.

For example  $47 \times 5$

Here 45 is an odd number, now  $47 - 1 = 46$ , half of this ( $46 / 2$ ) is 23 and by putting '5' behind that it become 235. Hence the answer is 235.



### **METHODS FOR MULTIPLICATION BY 25**

We know that  $25 = 100 / 4$ , hence to ease the computation, multiply the number by 100 (it is very simple just put two zeroes at the end of the number) and then divide the number by 4.

Lets take one example

$76 \times 25 = ?$

Now first multiply 76 by 100 i.e.  $76 \times 100 = 7600$

Now divide 7600 by 4 i.e.  $\frac{7600}{4} = 1900$ , hence the answer.



**MULTIPLICATION BY 12**

The method is exactly the same as in the case of 11 except that you double each number before adding the right neighbour.

(1)  $13 \times 12 = ?$

**Step 1:** Prefix a zero to the multiplicand so that it reads 013.

<b>Step 2:</b> Double 3 and add the right neighbour (there is no right neighbour; therefore add 0).	$013 \times 12$
Double 1 and add the right neighbour 3.	$\underline{\quad 6}$
$2 \times 1 + 3 = 5$	$013 \times 12$
Double 0 and add the right neighbour 1.	$\underline{\quad 56}$
$2 \times 0 + 1 = 1$	$013 \times 12$
	$\underline{156}$

Therefore,  $13 \times 12 = 156$  (which you can again verify by a conventional multiplication).

**MULTIPLICATION FROM 13 TO 19**

The reason why the rule is different for multiplication by 11 and by 12 is obviously because the right digits are different. The right digit, we could call the Parent Index Number (PIN). Thus in 11, the PIN is 1 and in 12 it is 2. (In 13, it is 3; in 14, it is 4 etc.) When the PIN is 1, we are simply taking each figure of the multiplicand (we could call this figure the Parent Figure – PF in short) as such and adding the right neighbour. When the PIN is 2, we are doubling the PF and then adding the right neighbour.

Obviously, if the PIN is 3 (as in 13), we would treble the PF and then add the right neighbour. If the PIN is 4 (as in 14), we would quadruple (i.e. multiply by 4) the PF and then add the right neighbour. If PIN is 9 (as in 19), we would multiply the PF by 9 and then add the right neighbour, what is the advantage of the method? We need know the tables only up to 9 and still multiply by a simple process of addition. *Let us see some examples below:*

(1)  $39942 \times 13 = ?$

2331
$039942 \times 13$
$\underline{519246}$

(2)  $43285 \times 14 = ?$

21132
$043285 \times 14$
$\underline{605990}$

(3)  $58265 \times 15 = ?$

34132
$058265 \times 15$
$\underline{873975}$

(4)  $36987 \times 16 = ?$

25654
$036987 \times 16$
$\underline{591792}$

**MULTIPLICATION OF TWO 2 DIGIT NUMBERS**

Consider the conventional multiplication of two 2-digit numbers 12 and 23 shown below:

$$\begin{array}{r} 12 \\ \times 23 \\ \hline 36 \\ 24 \\ \hline 276 \end{array}$$

It is obvious from the above that

- (1) the right digit 6 of the answer is the product obtained by the "vertical" multiplication of the right digit of the multiplicand and of the multiplier.
- (2) the left digit 2 of the answer is the product obtained by the "vertical" multiplication of the left digit of the multiplicand and of the multiplier.
- (3) the middle digit 7 of the answer is the sum of 3 and 4. The 3 is the product of the left digit of the multiplicand and the right digit of the multiplier; the 4 is the product of the right digit of the multiplicand and the left digit of the multiplier. This means that, to obtain the middle digit, one has to multiply "across" and add the two products (in our example  $1 \times 3 + 2 \times 2$ ).

The working in our above example can therefore be depicted as

$$\begin{array}{r} 12 \\ 23 \\ \hline 1 \times 2 / 1 \times 3 + 2 \times 2 / 2 \times 3 \\ \hline \text{and can be summarized as} \\ \hline 12 \\ 23 \\ \hline 2 / 3 + 4 / 6 = 276 \end{array}$$

**When the units figure is "one" in both the numbers** being multiplied, the process of multiplication is simplified further. Consider the following multiplication:

$$\begin{array}{r} 31 \\ 21 \\ \hline 2 \times 3 / 2 \times 1 + 1 \times 3 / 1 \times 1 \end{array}$$

You will notice that the middle digit of the answer is  $2 \times 1 + 1 \times 3$  i.e.  $(2 + 3) \times 1$ . So, instead of multiplying "across" for the middle term, you could simply add the tens digit of the two numbers.

Therefore,  $31 \times 21 = 6 / (2 + 3) / 1 = 651$ .

Similarly, in  $81 \times 91$ , you could obtain the middle term as  $1^7$  by merely adding 8 and 9.

Like wise, **when the tens figure is "one" in both the numbers** being multiplied, you could obtain the middle term by simply adding the units digit of the two numbers. For instance, the middle term in  $12 \times 17$  is  $2 + 7$ , i.e. 9, in  $8 \times 12$  it is  $8 + 2$  i.e. 10 etc.

If the **units figure or the tens figure is the same in the two numbers**, the process of multiplication could be simplified as shown in the *following examples*:

(1)  $\begin{array}{r} 8 \quad 3 \\ \underline{9 \quad 3} \\ 72519 \end{array} = 7719$  The middle term is obtained by multiplying 3 by 17, 17 being the sum of 8

(2)  $\begin{array}{r} 2 \quad 8 \\ \underline{2 \quad 3} \\ 4_2 2_2 4 \end{array} = 644$  In this case, the middle term is 2 multiplied by 11, 11 being the sum of 8.

**Here's a rather complicated trick for calculating any two 2-digit numbers:**

**Note:** To do this trick, you need to know the squares of numbers up to 100 in advance.

1. Calculate the average of the two numbers.
2. Square your answer (*here's where you need to know the squares of numbers through 100*).
3. Subtract the largest of your original two numbers from the average.
4. Square the answer.
5. Subtract it from your answer in step 2 for the final answer.

**Example:**  $27 \times 15$

First, calculate the average ( $42 / 2 = 21$ )

Then square it ( $21^2 = 441$ )

Now subtract largest number from the average ( $27 - 21 = 6$ )

Square your answer ( $6^2 = 36$ )

Subtract the square from the first square ( $441 - 36 = 405$ ). And there's your answer: 405

This works every time, but as the average and differences can end in .5, you have to know these squares as well.

By the way, this trick works because  $(x + y)(x - y) = x \times x - y \times y$

**MULTIPLICATION OF 2 THREE-DIGIT NUMBERS**

Let us consider the multiplicand to be ABC and the multiplier to be DEF, as shown below:

$$\begin{array}{r} A B C \\ \times D E F \\ \hline \text{Answer} \end{array}$$

1. The extreme right digit of the answer is obtained (as before) by vertical multiplication as  $C \times F$ .
2. The extreme left digit is also obtained (as before) by vertical multiplication as  $A \times B$ .
3. The "middle" digits are obtained (as before) by multiplying across. Progressing one step at a time to the left, the "middle" digits are successively

$$\begin{array}{l} B \times F + C \times E \\ A \times F + C \times D + B \times E \\ A \times E + B \times D \end{array}$$

The process is set out in detail for following examples below.

$$\begin{array}{r}
 1 \quad 2 \quad 3 \\
 4 \quad 5 \quad 6 \\
 \hline
 1 \times 4 / 1 \times 5 + 2 \times 4 / 1 \times 6 + 3 \times 4 + 2 \times 5 / 2 \times 6 + 3 \times 5 / 3 \times 6 \\
 \hline
 = 4/13/28/27/18 = 56088
 \end{array}$$

$$\begin{array}{r}
 2 \quad 4 \quad 5 \\
 1 \quad 9 \quad 8 \\
 \hline
 2 \times 1 / 2 \times 9 + 4 \times 1 / 2 \times 8 + 5 \times 1 + 4 \times 9 / 4 \times 8 + 5 \times 9 / 5 \times 8 \\
 \hline
 = 2/22/57/7/40 = 48510
 \end{array}$$

**MULTIPLICATION OF NUMBERS OF DIFFERENT LENGTHS**

In the examples we saw above, both the multiplicand and the multiplier contained the same number of digits. But what if the two numbers were to contain a different number of digits; for instance, how would we multiply 286 and 78? Obviously we could prefix a zero to 78 (so that it becomes 078, a 3 digit number) and proceed as in any multiplication of two 3 digit numbers.

The following examples will clarify the procedure:

$$\begin{array}{r}
 (1) \quad 286 \times 78 = ? \\
 \begin{array}{r}
 286 \\
 078 \\
 \hline
 01472 \quad 10648 \\
 \hline
 = 22308
 \end{array}
 \end{array}
 \qquad
 \begin{array}{r}
 (2) \quad 998 \times 98 = ? \\
 \begin{array}{r}
 998 \\
 098 \\
 \hline
 081153 \quad 14464 \\
 \hline
 = 97804
 \end{array}
 \end{array}$$

**TO MULTIPLY ANY TWO NUMBERS A AND B CLOSE TO A POWER OF 10**

- Take as base for the calculations that power of 10 which is nearest to the numbers to be multiplied.
- Put the two numbers **A** and **B** above and below on the left hand side.
- Subtract each of them from the base (nearest power of 10) and write down the remainders  $r_1$  and  $r_2$  on the right hand side either with a connecting minus sign between **A** &  $r_1$  and **B** &  $r_2$  if the numbers **A** and **B** are less than the closest power of 10. Otherwise, use a connecting plus sign between the numbers and the remainders.
- The final answer will have two parts. One on the left hand side and the other on the right hand side. The right hand side is the multiplication of the two remainders and the left hand side is either the difference of **A** and  $r_2$  or **B** and  $r_1$  if the numbers are less than the closest power of 10. Otherwise, it is the sum of **A** and  $r_2$  or **B** and  $r_1$

Few examples, which make the procedure clear, are:

**Multiplication of 9 and 7**

The closest base to the two numbers in this case is 10

Therefore  $9 - 1$  (The remainder after subtracting the number from 10)

$7 - 3$  (The remainder after subtracting the number from 10)

The right hand side of the answer will be  $1 \times 3 = 3$

The left hand side can be computed either by subtracting 3 from 9 or 1 from 7 which is 6. Therefore, the answer is 63.

**Numbers, which are close to 100**

The closest base in this case will be 100

$$\begin{array}{r} \text{Therefore} \quad 94 - 6 \\ \quad \quad \quad \underline{87 - 13} \\ \quad \quad \quad \underline{81 \ . 78} \rightarrow 8178 \end{array}$$

Here, 6 in the first row is the difference 100 and 94 and the 13 in the second row is the difference between 100 and 87. The right hand side of the answer is obtained by the multiplication of 6 and 13 which is 78 and the left hand side is obtained by the difference between either 87 and 6 or 94 and 13, both of which give the answer 81.

**Numbers which are greater than the closest power of 10**

Find the product of 108 and 112

The closest base is 100 in this case as well

$$\begin{array}{r} \text{Therefore} \quad 108 + 8 \\ \quad \quad \quad \underline{112 + 12} \\ \quad \quad \quad \underline{120 \ . 96} \end{array} \Rightarrow (120 + 0), 96 = 12096$$

The procedure is the same with only difference being that instead of subtracting the remainder of one number from the other number, we add in this case as the number were marginally larger than the nearest power of 10

When the number of digits of the product of the remainders is greater than the power of 10 closest to the two numbers

**E.g.** Product 84 and 92

$$\begin{array}{r} 84 - 16 \\ \underline{92 - 8} \\ \underline{76 \ . 28} \rightarrow (76 + 1), 28 \rightarrow 7728 \end{array}$$

As the product of 16 and 8 is 128 which is a three digit number as against 2 being the power of 10 in 100, we carry forward the digits on the left more than 2 digits (in this case) and add to 76, the left hand side of the answer

**E.g. When one of the number is lesser than the closest power of 10 and the other greater than the closest power of 10.**

$$\begin{array}{r} \text{Product of 88 and 106} \\ 88 - 12 \\ \underline{108 + 8} \\ \underline{96 / 96} \end{array}$$

The operation is similar, excepting that as the right hand side of the answer is obtained by the multiplication of a positive and a negative number the answer has to be subtracted from 100 by reducing the left hand side number by 1. So the answer is 9504.



**Multiplication of numbers which are not close to the nearest power of 10**

Let us take the case of multiplication of 41 by 43. Going by the earlier method we have the nearest power of 10 as 100 or 10. In the former case, The remainders are 59 and 57, multiplication of which will be as tedious as the multiplication of these two numbers. In the latter case, The remainders will be 31 and 33 which will be equally difficult. Therefore, we need to look at an alternative method. In this case, we can take 50, which is a sub multiple of 100 or a multiple of 10 and proceed

**Method 1: Take 50 as the base which is half of 100**

$$41 - 9$$

$$43 - 7$$

$34 / 63 \rightarrow$  since  $50 = 100/2$ . We divide the left hand side number also by 2 while retaining the right hand side. Therefore the answer will be 1763.

**Method 2: We can start off instead of using 50 as the base, we can use 40 as the base**

$$41 + 1$$

$$43 + 3$$

$44 / 3 \rightarrow$  and since 40 is 4 times 10, we multiply 44 by 4 to yield by 176 and join the right hand side to yield 1763 – the same answer.

**MULTIPLICATION OF ANY THREE DIGIT NUMBER WITH 101**

For example

$$349 \times 101$$

Add 3 and 9. This is 12. 2 is middle digit of answer, 1 is carry forward.

Take first two digits i.e. 34. Add carry forward, 1 to it. This is 35. These are first two digits of answer.

Take last two digits of number, 49. 49 is last two digits of answer.

Hence answer is 35249.

Lets take few more examples

1.  $981 \times 101$

a)  $9 + 1 = 10 \_ \_ 0 \_ \_$

b)  $98 + 1 = 99 \ 990 \_ \_$

c) 81

Hence 99081 is the answer.

2.  $477 \times 101$

a)  $4 + 7 = 11 \_ \_ 1 \_ \_$

b)  $47 + 1 = 48 \ 481 \_ \_$

c) 77

Hence 48177 is the answer.

**MULTIPLY THREE DIGIT NUMBER WITH  $7 \times 11 \times 13$**

What's the answer?

Simply write the number twice.

e.g.  $346 \times 7 \times 11 \times 13 = 346346$

$845 \times 7 \times 11 \times 13 = 845845$

**PRODUCT OF NOS. (ENDING WITH 5) WHICH DIFFER BY 10**

E.g.  $45 \times 35$

**Step 1:** First of all write 75 in the last two places of the product  $45 \times 35 = \underline{\quad}75$

**Step 2:** Then multiply 4 with 3 and add the smallest of these to the product i.e.  $(4 \times 3 + 3) = 15$

**Step 3:** Now Place these two digits before 75 to get our required product i.e.  $45 \times 35 = 1575$

**Product of two complementary numbers**

**What are they?**

Before we begin the discussion on how to calculate the product, we will first define what we mean by complementary numbers.

Complementary numbers are two numbers that add up to a power of 10. For example, 2 and 8 are complementary, because they add up to 10. Also 22 and 78 are complementary, because they add up to 100, and so on.

**Numbers that add up to 10**

Almost everyone would be expected to calculate  $2 \times 8$  (or  $2 \times 8$ ) through the memorisation of the multiplication table. A less well known fact is that  $2 \times 8$  equals the difference of the square of 5 (average of two and eight) and the square of 3 ( $= 8 - 5 = 5 - 2$ ), i.e.  $2 \times 8 = 5 \times 5 - 3 \times 3 = 16$ .

**Numbers that add up to 100, 1000, 10000 etc.**

To calculate the product of  $42 \times 58$ , we make use of the relationship described in the previous paragraph:

$$42 \times 58 = (50 - 8) \times (50 + 8) = 50 \times 50 - 8 \times 8 = 2500 - 64 = 2436.$$

In other words, to multiply two numbers  $N_1$  and  $N_2$ , we just have to square the average number  $N$ , ( $= 5, 50, 500\dots$ ) and subtract from it the square of the difference  $(N - N_1)$ .

$$N_1 \times N_2 = N \times N - (N_2 - N) \times (N - N_1).$$

**Examples:**

$$45 \times 55 = 50 \times 50 - 5 \times 5 = 2500 - 25 = 2475.$$

$$4899 \times 5101 = 5000 \times 5000 - 101 \times 101 = 25,000,000 - 10201 = 24,989,799$$

**Product of two partially complementary numbers**

**What are partially complementary numbers?**

We have seen that complementary numbers add up to a power of 10. Partially complementary numbers have their first digits common (the root), and their last digits (suffix) add up to a power of 10. For example, 43 and 57 are complementary, because 43 and 57 add up to 100, a power of 10. On the other hand, 343 and 357 are partially complementary, the root is 3 and the suffices are 43 and 57.

**Suffices that add up to 10**

To calculate the product, you would have to calculate product of the root and the next number, and append the product of the suffices.

To multiply 53 by 57, we identify that the root is 5 and the suffices are 3 and 7. The product of the root and the next number is therefore  $5 \times (5 + 1) = 30$ . Appending the product of 3 and 7 ( $= 21$ ) to 30 gives 3021.

$$\text{Thus } 53 \times 57 = 3021.$$

Note that this is almost the same procedure as the calculation of squares of numbers ending in 5. Instead of appending 25, we append the product of the suffices.

- Consider (i)  $2\bar{3} \times 2\bar{7}$   
 (ii)  $9\bar{4} \times 9\bar{6}$   
 (iii)  $98\bar{2} \times 98\bar{8}$

In all of them, the units figures together total 10 [3 + 7 in (i), 4 + 6 in (ii) and 2 + 8 in (iii)], also the other figures of the multiplicand and the multiplier are the same [2 in (i) 9 in (ii) and 98 in (iii)].

In such cases, a special method can be used. But before we see the new method, let us consider the multiplication of 23 by 27 by the method we had learnt.  $23 \times 27$  would give us the answer as 621. Do you notice anything special about the answer? The right part is the product of the unit figures 3 and 7 of the 2 numbers and the left part 6 of the answer is the product of 2 (the tens figure) and the next higher number 3.

**This gives us the special rule:**

- (1) To obtain the right part of the answer, multiply the units (i.e. of the extreme right) digit of the 2 numbers.
- (2) To obtain the left part of the answer, multiply the other (i.e. the left), digit/s by one more than itself/themselves. For example, if the left digit/s of the 2 numbers is/are

- 3, multiply 3 by (3 + 1) i.e. by 4
- 5, multiply 5 by 6
- 6, multiply 6 by 7
- 99, multiply 99 by 100
- 100, multiply 100 by 101
- 888, multiply 888 by 889 etc

The only thing you have to be careful about is to ensure that the right part of the answer always has 2 digits. For example in  $29 \times 21$ , the left part of the answer is 2 (2 + 1) and the right part is  $9 \times 1$  i.e. 9; however the right part will be written as 09 and not as 9 because the right part has to contain 2 digits. Therefore,  $29 \times 21 = 609$ .

**Suffices that add up to 100, 1000, 10000 etc.**

The same steps will be followed. Multiply the root by the next number. Append the product of the suffices. For example, for  $1545 \times 1555$ , the root is 15, suffices are 45 and 55. Thus the product of the root and the next number is  $15 \times (15 + 1) = 240$ . To this, we append the product of the suffices,  $45 \times 55 = 2475$ . The suffices here are two digits long, and their product must therefore occupy four digits. Thus  $1545 \times 1555 = 2402475$ .

**MULTIPLICATION OF NUMBERS WHICH ARE CLOSE TO EACH OTHER**

- $24 \times 26 = ?$
- $28 \times 32 = ?$
- $87 \times 93 = ?$

In this shortcut we use the application of algebra. we know  $a^2 - b^2 = (a + b)(a - b)$

Now as  $24 \times 26$  can be written like  $(25 + 1) \times (25 - 1)$ ,

Now using above formula it is equal to  $25^2 - 1^2 = 625 - 1 = 624$ , which is the required answer.

Similarly

$87 \times 93 = (90 + 3) \times (90 - 3) = 90^2 - 3^2 = 8100 - 9 = 8091$

$28 \times 32 = (30 + 2) \times (30 - 2) = 30^2 - 2^2 = 900 - 4 = 896$

SQUARES OF NUMBERS

It is often useful to know the squares of numbers without a calculator. For example, you have to paint the ceiling of a room of 12'-6" × 12'-6". How much paint would you buy if a quart covers 8 square metres? You would reason as follows:

A quart covers 8 square metres = 80 sq.ft.

$12.5 \times 12.5 = 156.25$ , therefore you'll need 2 quarts of paint to cover 160 sq.ft.

In general, it is quite possible that an average person would be able to give the squares of integers up to 100 without a calculator. Some of the numbers are easily memorized, and the others can be obtained by certain 'tricks'. This article provides some tools to memorize or calculate the square of some numbers.

Numbers	Method
1 – 25	Memorization
Numbers ending in a five 15, 25, 35, 45, ...	Remove the last digit (five), multiply the resulting number (n) by the next number (n + 1), and tag on a 25 at the end of the product. <b>Example:</b> $45 \times 45$ $4 \times 5 = 20$ Tag on a 25 to make 2025. Calculate $45 \times 45 = 2025$
25 – 50	Calculate the difference (d) from 50. Subtract $100 \times d$ from 2500. Add $d \times d$ to the result. <b>Example:</b> To calculate $47 \times 47$ : $d = 3$ Calculate $47 \times 47 = 2500 - 300 + 9 = 2209$
50 – 75	Subtract 50 from the number to give d. Add $100 \times d$ to 2500. Add $d \times d$ to the result. <b>Example:</b> To calculate $53 \times 53$ : $d = 3$ Calculate $53 \times 53 = 2500 + 300 + 9 = 2809$
75 – 100	Subtract the number (n) from 100 to give d. Calculate $(n - d) \times 100 + d \times d$ <b>Example:</b> To calculate $96 \times 96$ : $d = 4$ Calculate $96 \times 96 = (96 - 4) \times 100 + 4 \times 4 = 9200 + 16 = 9216$ .

$100 - 125$	<p>Subtract 100 from the number (n) to give d.                  Calculate <math>(n + d) \times 100 + d \times d</math>  <b>Example:</b>                  To calculate <math>108 \times 108</math>:  <math>d = 8</math>                  Calculate <math>108 \times 108 = (108 + 8) \times 100 + 8 \times 8 = 116 \times 100 + 64 = 11664</math>.</p>
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**Some other Methods of Squaring**

**1. Let's start with numbers 11 – 19.**

I know, I know, 11 and 12 shouldn't pose any problem but anything higher than 13 was never a flashcard...

Square the ones digit. This becomes your last digit.

1. Multiply the ones digit by 2. This becomes your second digit.
2. Retain the tens digit (i.e., 1) and append the above results.

Take  $17^2$ , for example.

$7^2 = 49$ . The 9 is your last digit. Carry the 4 to the left.

$7 \times 2 = 14$ . The 4 (plus the 4 remaining from the first step) is your second digit, 8. As before carry the 1 to the left.

Retain the 1 from "17", add to it the 1 remaining from the previous step, and you get 289.

Interesting but probably not as quick as simply multiplying the numbers.

**2. General Method of squaring numbers 51 to 100:**

1. Subtract the number you want to square from 100.  $(100 - x)$
2. Take this difference and subtract it from the number you want to square.  $(x - (100 - x))$ . This is the first 2 digits of your answer.
3. Square the difference, the answer from step 1. This is the last 2 digits of your answer.

Take  $96^2$ , for example.

$100 - 96 = 4$ .

$96 - 4 = 92$ . You're halfway there. 92\_\_.

$4^2 = 16$ . There's your last 2 digits. Thus, your answer: 9,216.

Try  $88^2$ .

$100 - 88 = 12$

$88 - 12 = 76$ . 76\_\_.

$12^2 = 144$ . What now?  $88^2$  isn't 76144!  $92^2$  was only 9216.

OK enough suspense. Just carry the 1 over the 6.

\_144

76\_\_

7,744

**If you like that, well check this out for numbers 50-59:**

It's a simple rule: Square the tens digit and add to that the ones digit of your original number. That's your first two numbers. Second, square your ones digit. That's your last two numbers.

Get it? Try it with 54. It should make sense.

1.  $5^2 + 4 = 29$ . That's our first two numbers.
2.  $4^2 = 16$ . That's our last two numbers. Thus, the answer 2916.

**Another Method for Squaring numbers form 51 to 59.**

**There is one general formula**

$$(5x)^2 = (25 + x)/x^2$$

Where x is a digit in units place

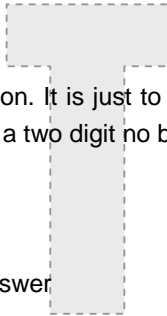
**Note:** Slash here does not represent division. It is just to differentiate between the two value.  $x^2$  should be a two digit no. If it is a single digit no, make it a two digit no by placing a prefix 0 before  $x^2$

**E.g.**  $3^2 = 9$ , Make it of two digits i.e. 09

Let us calculate the squares of 54

In this case our x is 4

$$(54)^2 = (25 + 4)/4^2 = 29/16 = 2916 \text{ is our answer}$$



**What about numbers greater than 100?**

Here are the steps and an example.

1. Subtract 100 from the number you want to square ( $x - 100$ ).
2. Take the difference and add it to the number you want to square. ( $x + (x - 100)$ ). This is the first 3 digits of your answer.
3. Square the difference, the answer from step 1. This is the last 2 digits of your answer.

**Example:  $112^2$**

$$112 - 100 = 12$$

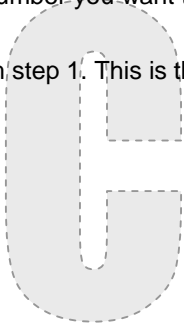
$$112 + 12 = 124. 124\_\_\_$$

$$12^2 = 144. \text{ Carry the 1 over the 4.}$$

$$\_\_144$$

$$124\_\_\_$$

$$12,544$$



**What about numbers around 1000?**

Use the same rules above. Here are 2 examples:

$$996^2$$

$$1000 - 996 = 4$$

$$996 - 4 = 992. 992\_\_\_ \text{ First 3 digits.}$$

$$4^2 = 16. \text{ Last 3 digits.}$$

**Answer: 992,016**

Make sense?

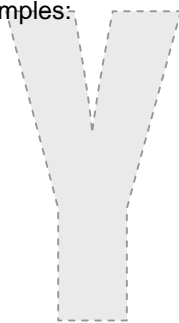
$$1007^2$$

$$1007 - 1000 = 7$$

$$1007 + 7 = 1014. 1014\_\_\_ \text{ First 4 digits.}$$

$$7^2 = 49. \text{ Last 3 digits.}$$

**Answer: 1,014,049**



**Another Method for squaring any two digit number**

"a" is a digit in tens place. It is replaced by digits from 1 – 9.

**Procedures:**

	The number being subtracted	Number subtracted (From ten's place digit)
Ending	1 a(a + 1)01	8a
	2 a(a + 1)04	6a
	3 a(a + 1)09	4a
	4 a(a + 1)16	2a
	5 a(a + 1)25	
	The number being added	Number added (To ten's place digit)
	6 a(a + 1)36	2a
	7 a(a + 1)49	4a
	8 a(a + 1)64	6a
9 a(a + 1)81	8a	

**Note:** The numbers that are stacked to a(a + 1) are easily found by squaring the digit in the ones place. a1, (1<sup>2</sup>) then 01 is the number that will stack. a2, (2<sup>2</sup>) then 04 is the number that will stack. a3, (3<sup>2</sup>) then 09 is the number that will stack. a4, (4<sup>2</sup>) then 16 (**Not** 016) is the number that will stack. a5, 25; a6, 36; and so on.. up to a9 in which 81 will be the number to stack on a(a + 1).

**Example:**

Ending	1	11 × 11	1(1 + 1)01 = 201, subtract 8 from ten's place(20 – 8) to get 121
	2	12 × 12	1(1 + 1)04 = 204, subtract 6 from ten's place(20 – 6) to get 144
	3	13 × 13	1(1 + 1)09 = 209, subtract 4 from ten's place(20 – 4) to get 169
	4	14 × 14	1(1 + 1)16 = 216, subtract 2 from ten's place(21 – 2) to get 196
	5	15 × 15	1(1 + 1)25 = 225
	6	16 × 16	1(1 + 1)36 = 236, Add 2 to ten's place(23 + 2) to get 256
	7	17 × 17	1(1 + 1)49 = 249, Add 4 to ten's place(24 + 4) to get 289
	8	18 × 18	1(1 + 1)64 = 264, Add 6 to ten's place(26 + 6) to get 324
	9	19 × 19	1(1 + 1)81 = 281, Add 8 to ten's place(28 + 8) to get 361

**How do we remember what number to add or subtract ?**

If the ending digit of your two digit number is between 1 – 4, the operation is subtraction. To know what number to subtract from the ten's digit, **add the digits in one's place and subtract the result from ten.** [Don't forget to multiply the last result by "a"].

If the ending digit is between 6 – 9, the operation is addition. To know what number to add to the ten's digit, **add the digits in one's place and subtract ten from the result.** [Don't forget to multiply the last result by "a"]

Remember, use this method only for the square of two digit numbers.

**SQUARING NUMBERS ENDING IN 5**

An obvious extension of the above method will be in finding the square of numbers ending in 5. Consider  $15 \times 15$ . Here the sum of the units digit of the 2 numbers is the same. Hence, the above method will apply. Infact it will apply in squaring any number ending in 5. Since in all these cases the right digit of the multiplicand and the multiplier is 5, the right part of the answer is always 25 and therefore, we can mechanically set down 25 as the right part of the answer without any calculation – all that is needed is to find out the left part of the answer and this done exactly as in the previous section:

$15^2 = 225$        $25^2 = 625$        $875^2 = 765625$        $8995^2 = 80910025$

**SQUARING NUMBERS ONLY ONES**

$(11)^2 = ?$

$(111)^2 = ?$

$(1111)^2 = ?$

It is very simple just count the number of '1' in the question and write the natural numbers up to that number in ascending order then in descending order up to 1. So the answer for the above questions are

$(11)^2 = 121$

$(111)^2 = 12321$

$(1111)^2 = 1234321$

**SQUARING NUMBERS ONLY NINES**

$(99)^2 = ?$

$(999)^2 = ?$

$(9999)^2 = ?$

In this method, count the number of nines in the question.

Let there are n number of nines. Then square of the number is

$(n - 1)$  nines | 8 |  $(n - 1)$  zeroes | 1

$(99)^2 =$  two nines i.e.  $n = 2$ , hence the answer is  $\Rightarrow$   $(2 - 1)$  nines | 8 |  $(2 - 1)$  zeroes | 1 = 9801

$(999)^2 =$  here  $n = 3$ , hence the answer is  $\Rightarrow$  two nines | 8 | two zeroes | 1 = 998001

$(9999)^2 =$  here  $n = 4$ , hence the answer is  $\Rightarrow$  99980001



**SQUARING OF NUMBER BETWEEN 101 AND 199**

For example  $(119)^2 = ?$

- a) Add last two digit of number to the number  $119 + 19 = 138$
- b) Square last two digits  $19 \times 19 = 361$   
Take last two digits, 61 as last two digit of answer. Rest of the numbers are carry forwards.
- c) Add carry forward, 3 to 138.  $138 + 3 = 141$   
These are first three digits of answer.  
So answer is 14161.

Take few more examples.

- 1.  $(145)^2 = ?$ 
  - a)  $145 + 45 = 190$
  - b)  $45 \times 45 = 2025$
  - c)  $190 + 20 = 210$

Hence the answer is, 21025.

- 2.  $106 \times 106$ 
  - a)  $106 + 06 = 112$
  - b)  $06 \times 06 = 36$
  - c)  $112 + 0 = 112$

Hence the answer is, 11236.

CUBE ROOTS OF EXACT CUBES

- (1) The lowest cubes, i.e. the cubes of the first nine natural numbers are 1, 8, 27, 64, 125, 216, 343, 512 and 729.
- (2) Thus, they all have their own distinct endings; and there is no possibility of overlapping or doubt as in the case of squares.
- (3) Therefore, the last digit of the cube root of an exact cube is obvious:
 

(i) Cube ends in 1;	$\therefore$	cube root ends in 1;
(ii) Cube ends in 8;	$\therefore$	cube root ends in 8;
(iii) Cube ends in 3;	$\therefore$	cube root ends in 7;
(iv) Cube ends in 4;	$\therefore$	cube root ends in 4;
(v) Cube ends in 5;	$\therefore$	cube root ends in 5;
(vi) Cube ends in 6;	$\therefore$	cube root ends in 6;
(vii) Cube ends in 7;	$\therefore$	cube root ends in 3;
(viii) Cube ends in 8;	$\therefore$	cube root ends in 2; and
(ix) Cube ends in 9;	$\therefore$	cube root ends in 9;
- (4) In other words,
  - (i) 1, 4, 5, 6, 9 and 0 repeat themselves in the cube-ending; and
  - (ii) 2, 3, 7 and 8 have an inter-play of complements from 10.
- (5) The number of digits in a cube root is the same as the number of 3-digit groups in the original cube including a single digit or a double-digit group if there is any:
- (6) The first digit of the cube-root will always be obvious from the first group in the cube.

- (7) Thus, the number of digits, the first digit and the last digit of the cube root of an exact cube are the data with which we start, when we begin the work of extracting the cube root of an exact cube.

**To instantly determine the cube root, follow these easy steps:**

Example: Find the cube root of 438976.

1. Drop the last three digits and find the largest cube contained in 438. This is  $7^3 = 343$ , so the ten's-digit is 7. (This is why you had to memorize the cubes of the digits 1 through 9).
2. Now go back to the last three digits. Look at the last digit, 6. That's the same ending as  $6^3$ , so your unit's digit is 6. So the cube root of 438976 is 76.

**Another example:**

How do you instantly determine the cube root of 79507?

1. Drop the last three digits and find the largest cube in 79. This is  $4^3 = 64$ , so the ten's-digit of the cube root is 4.
2. Now go back to the last three digits. Look at the last digit, 7. That has the same ending as 3-cubed. So the unit's-digit of your cube root is 3.  
Therefore, the cube root of 79507 is 43.

**APPROXIMATIONS OF SQUARE ROOTS TO TWO DECIMAL PLACES**

If you have to calculate the square root of 90 fairly accurately, how do you do this quickly? Follow this thumb rule:

- (1) Find the integer whose square is nearest to 90.  
(We can take 9, as  $9^2 = 81$  is close to 90)
- (2) Subtract the square of this root from the number, i.e., 90.

This means 90

$$\begin{array}{r} (-) \frac{81}{9} \end{array}$$

- (3) Divide the resulting number by twice the value of the square root (i.e., 9)

$$\text{or, } \frac{9}{9 \times 2} = 0.5$$

This is the correction to be made to the square root approximated earlier.

Add this value to the root estimated and get the approximate value of the square root, i.e.  $9 + 0.5 = 9.5$ .

Therefore, 9.5 is the approximate square root of 90.

**Another example:**

- (1) Square root of 250  
Nearest square is 256  
Estimated square root is 16  
250 minus 256 = - 6

$$\text{Divide this by 2 times 16} = \frac{-6}{32} = -0.1875$$

$$\text{Add to 16} = 16 - 0.1875 = 15.8125$$

**Another method (Clay's Method) for finding out approximate square root**

If you're good with long division, here's a quick way to find pretty accurate square roots without the aid of a calculator. Let's try 24.6.

1. Make a guess. It can be a very bad guess. It doesn't matter. You can even guess 1. Let's try 5 since  $5^2$  is 25, which is pretty close to 24.6.
2. Divide 24.6 by 5.  $24.6 / 5 = 4.92$
3. Now, comes the trick: **Pick a new guess between 5 and 4.92 and divide it into 24.6 again.** Let's try 4.95.  $24.6 / 4.95 = 4.96$ . 4.96 is pretty close to 4.9598 which is the actual square root of 24.6.
4. Repeat steps 2 and 3 to any desired level of accuracy. The further you go, the harder the long division becomes. But the first few cycles yield a pretty close answer.

The reason this works is because  $n \cdot n = 24.6$  and  $n = 24.6 / n$ . Therefore, the real square root will always be somewhere between  $24.6 / n$  and  $n$ .

**Another method (Newton's method) to find the square root of a number.**

Let's try to calculate the square root of 24.6. As Clay did, the initial estimate is 5. So  $N = 24.6$ , and  $E = 5$ .

The Newton's method says a better estimate,  $E_1$  is  $E_1 = E + (N - E \times E)/(2E)$ .

$$= 5 + (24.6 - 5 \times 5)/(2 \times 5)$$

$$= 5 - 0.4 / 10$$

$$= 4.96 \text{ (error} = 0.0033\%)$$

This can easily be handled using mental arithmetic, and is pretty close to the real value of 4.9598. Continuing to use mental arithmetic (see Clay's article on multiplication tricks), we can mentally calculate  $4.96 \times 4.96 = 5 \times 5 - 2 \times 0.04 \times 5 + 0.0016 = 24.6016$ .

This time the residue is 0.0016, divided by  $2 \times 4.96 = 9.92$  gives the next estimate,

$$E_2 = E_1 + (N - E_1 \times E_1) / (2 \times E_1)$$

$$= 4.96 - .0016/9.92,$$

Now, we can look at how we can handle the division. The approximation  $0.0016/10 = 0.00016$  is a very close. In fact, it gives  $E_2 = 4.95984$ , the error is 0.0000261%!

However, if you want to push your mental arithmetic, the division mentally is possible! You notice that dividing by 9.92 instead of 10 gives 0.8% more to the result, which we can simply add to  $0.00016 + 0.00000128 = 0.00016128$

Therefore

$$E_2 = E_1 - 0.00016128 = 4.95983872, \text{ compared to the actual value of } 4.959838707 \text{ (error of } 0.0000003\%).$$

I think we fare better than calculators displaying up to 8 digits, using just our brains, not even pencil and paper!

COMPARISON OF FRACTIONS

We know that, if denominator of the all fractions is same, then fraction having bigger numerator is bigger in value and having smaller numerator is smaller in value.

Let's take one example.

Out of  $\frac{3}{11}$ ,  $\frac{4}{11}$ ,  $\frac{7}{11}$ , the first fraction is the smallest and the last one is the biggest.

Also if the numerator of all fractions is same, then the fraction having bigger denominator is smaller in value and vice versa.

Let's take one example.

Out of  $\frac{13}{4}$ ,  $\frac{13}{5}$ ,  $\frac{13}{6}$ , the first fraction is the biggest and the last one is the smallest.

What happens when if neither numerator nor denominator of the fractions is same. We know in this case, basic method is by LCM method, i.e. making the denominator same. But we don't know that we can even make the numerator same. So we have to check that whose LCM is easy to find out, depending upon the value of the numerators or the denominators.

Let's understand it with the help of an example.

Compare  $\frac{1}{25}$ ,  $\frac{2}{35}$ ,  $\frac{3}{30}$

Now we can see that finding out of the LCM of the 25, 35 and 30 is somewhat tedious. So we find out the LCM of the numerators, i.e. LCM of 1, 2 and 3. Which is 6. So our problem reduced to comparison of  $\frac{6}{150}$ ,  $\frac{6}{105}$ ,  $\frac{6}{60}$ . Here we can check that being the numerator same, last fraction is the biggest.

**Comparison of the fractions, when they are in certain fashion.**

When relationship between each of the numerator and denominator is same, difference between them is same and with same sign. It has two cases.

1. If each numerator is smaller than its denominator by same value. In this situation the fraction having bigger digits will be bigger in value and vice versa.

Let's take one example:

Compare  $\frac{2}{3}$ ,  $\frac{4}{5}$ ,  $\frac{5}{6}$ ,  $\frac{6}{7}$

Using above method we can easily compare all of them. Hence  $\frac{6}{7}$  is the biggest,  $\frac{5}{6}$  is second biggest,

$\frac{4}{5}$  is third biggest and  $\frac{2}{3}$  is the smallest.

2. If each numerator is greater than its denominator by same value. In this situation the fraction having bigger digits will be smaller and vice versa.

Let's take one example:

Compare  $\frac{13}{11}$ ,  $\frac{15}{13}$ ,  $\frac{19}{17}$

Using above method we can easily say  $\frac{13}{11}$  is the biggest,  $\frac{15}{13}$  is second biggest and  $\frac{19}{17}$  is the smallest.

**Logical Comparison**

This shortcut more depends upon our logic. The basic requirement to use this method is, we should be able to calculate the approximate value of the fraction.

Let's take one example

Compare  $\frac{15}{28}$ ,  $\frac{16}{34}$

Now look at the first fraction, we see that denominator is 28 and half of the 28 is 14, 15 is greater than 14, hence value of this fraction is slightly more than  $\frac{1}{2}$ .

Similarly in second fraction, denominator is 34 and half of 34 is 17, 16 is less than 17, hence value of this fraction is slightly less than  $\frac{1}{2}$ .

Combining these two results we can judge that  $\frac{15}{28}$  is greater than  $\frac{16}{34}$ .

**PERCENTAGE**

**Basic Shortcut**

The most basic tip for the questions involving percentage is memorize the fractional equivalent of the percentage. i.e.

$25\% = \frac{25}{100} = \frac{1}{4}$

$20\% = \frac{20}{100} = \frac{1}{5}$

$10\% = \frac{10}{100} = \frac{1}{10}$

Similarly, we can generate lot more.

$50\% = \frac{1}{2}$

$40\% = \frac{2}{5}$

$60\% = \frac{3}{5}$

$75\% = \frac{3}{4}$

$5\% = \frac{1}{20}$  etc

This means when we have to calculate 25 % of 50, we can directly calculate one fourth of 50, which is 12.5 Similarly if we have to calculate 5 % of 400, we calculate one twentieth (1/20) of the 400, which is 20.

**Another shortcut**

You may find it difficult to compute 18% of 25. Instead, try solving for 25% of 18. 25% is just half of half of any number. Half of 18 is 9, and half of 9 is 4.5.

Since 25% of 18 is 4.5, it is also true that 18% of 25 is 4.5.

This is something our teachers never taught us in school, and it would have helped a lot in everyday life --  $A\%$  of  $B = B\%$  of  $A$ .

Like multiplication, percentages conform to the commutative principle, which says the order of the terms does not matter. (Just as  $2 \times 3 = 3 \times 2$ , it is also true that  $40\%$  of  $5 = 5\%$  of  $40$ ).

